Reciprocal Green’s Functions and the Quick Forecast of Submarine Landslide Tsunami

Guan-Yu Chen1, Chin-Chih Liu1, Janaka J. Wijetunge2, Yi-Fung Wang3

1Department of Oceanography, National Sun Yat-sen University, Kaohsiung 80424, Taiwan
2Faculty of Engineering, University of Peradeniya, Peradeniya 20400, Sri Lanka
3Water Resources Agency, MOEA, Taipei 10651, Taiwan

Correspondence to: Guan-Yu Chen (guanyu@faculty.nsysu.edu.tw)

Abstract. Although tsunamis generated by submarine mass failure are not as common as those induced by submarine earthquakes, sometimes the generated tsunamis are higher than a seismic tsunami in the area close to the tsunami source, and the forecast is much more difficult. In the present study, reciprocal Green’s functions are proposed as a useful tool in the forecast of submarine landslide tsunamis. The forcing of the continuity equation due to depth change in a landslide is represented by the temporal derivative of the water depth. After a convolution with the reciprocal Green’s function, the tsunami waveform can be obtained promptly. Thus, various tsunami scenarios can be considered once a submarine landslide happens, and a useful forecast can be formulated. When a submarine landslide occurs, the various possibilities for tsunami generation can be analysed, and a useful forecast can be devised.

1 Introduction

A Tsunami is a serious hazard to coastal cities and its forecast is essential for hazard mitigation. Of all tsunami hazards, seismic tsunamis are easier to forecast because earthquake information can be retrieved and broadcast very quickly. With the aid of elasticity theories and regression formula for assessing the length scale of fault ruptures, the tsunami source can be estimated with satisfactory accuracy. Based on Green’s function (GF; see, e.g. Wei et al., 2003), reciprocal Green’s function (RGF; see, e.g. Chen et al., 2012), or real-time direct simulation, the propagation of tsunami is calculated in a short time. The coastal inundation then can be obtained by real-time direct simulations, analytical solutions (see, e.g., Lin et al. 2014), or pre-calculated inundation maps (see, e.g., Chen et al., 2015). Some of the approaches mentioned above have been integrated and an economical forecast system has been developed to provide both offshore water surface elevation and an inundation map (Chen et al. 2015). The efficiency and robustness of these systematic analyses are superior to real-time equation-solving, as has been shown in previous studies.

Besides seismic tsunamis, a few recent events are believed to be closely related to submarine mass failure (SMF). For example, the 1998 Papua New Guinea Tsunami (Tappin et al., 2001), the 2007 Chilean Tsunami (Sepúlveda et al., 2010) and the 2018 Sulawesi Tsunami (Heidarzadeh et al., 2019) all occurred after submarine earthquakes. However, in each case the earthquake was not strong enough to generate a big tsunami. The devastating tsunamis following the earthquake were all
attributed to submarine landslides triggered by the earthquake. Another rare event in the Sunda Strait, 2018, was also generated by a submarine landslide triggered by volcanic eruption (Wikipedia website).

Besides these recent events, some historical events are also believed to be the result of submarine mass failure (SMF). The mysterious tsunami that struck the southwest coast of Taiwan (Li et al., 2015) is an example, which will be simulated later as an example in the present study.

Although the RGF approach is quick and economical, extending this approach from seismic tsunami to SMF tsunami is not straightforward: Fault rupture in an earthquake is much faster than the water wave speed and hence the rupture process can be simply represented by initial sea surface elevations which are determined by sea bottom deformation after the fault rupture. Thus, only the response to initial water level is needed. On the other hand, an SMF forces the sea water and continuously contributes to the formation of a tsunami; a much more complicated computation is involved.

As SMF tsunami have been devastating to coastal areas, its forecast and associated hazard mitigation are very important. However, no available technology can provide accurate information on the details of the SMF. The location, depth, volume, density, directional movement, movement speed, distance and duration of the slide displacement are difficult to determine accurately. As the RGF approach is fast and robust, different SMF parameters and locations can be considered very quickly. Thus, ensemble forecasting of SMF tsunami becomes possible and can be used for tsunami hazard mitigation.

2 Research Method

Mathematically, two equation sets will be presented: One is the shallow water equations (SWEs) with SMF forcing. The other is SWEs with impulsive forcing represented by a delta function. With the aid of an integral transform, the complete solution to the SWEs is a convolution of the forcing and the GF. The detailed mathematics and its physical meaning will be presented in this section.

Because of the reciprocity between GF and RGF, the SMF tsunami can be obtained as the convolution of the slide forcing and the RGF. This convolution approach with RGF is then applied to a few idealized SMF scenarios and the results are compared in the next section with direct simulation using the Cornell Multigrid Coupled Tsunami Model (COMCOT). These comparisons are used to verify the RGF-convolution approach in calculating SMF tsunamis.

2.1 Green’s Functions for Shallow Water Equations

GFs are responses of a system to an impulsive point forcing. For homogeneous medium with infinite domain, GFs can be obtained analytically. The distribution of analytic GFs for various differential equations can be obtained from boundary conditions by numerical approaches such as the boundary element method (see, e.g., Brebbia et al., 1984).

Another type of GF includes both the inhomogeneity and the boundary conditions. In this case, an analytic solution is usually not available and each GF has to be solved numerically. Numerical GFs have previously been applied in seismic tsunami forecast (e.g., Wei et al., 2003). The good news is, this kind of forecast can be completed in a very short time if the
GFs are pre-calculated. There will be no need for equation-solving, and the forecast is simply a summation over the product of the initial sea surface elevations and the corresponding GFs.

The physical meaning of GF for shallow water equations (SWEs) is explained as follows. By definition mass flux is the integration of a velocity component from sea bottom to water surface, and equals the average velocity multiplied by the undisturbed water depth d. The mass flux vector and horizontal gradient operator are defined for brevity as

\[ \vec{V} = (P, Q) \]

and

\[ \nabla_h = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \]

where \( P \) and \( Q \) are respectively mass fluxes in two horizontal directions \( x \) and \( y \). If an impulsive forcing (delta function) is added on the right hand side of the continuity equation, SWEs become

\[ \frac{\partial}{\partial t} \eta + \nabla_h \cdot \vec{V} = \delta(t)(x-x_s)\delta(y-y_s) \]

\[ \frac{\partial}{\partial t} \vec{V} + gd \nabla_h \eta = 0 \]

where \( \eta \) is sea surface elevation and \( d \) is the undisturbed water depth. Integrating the continuity eq. over an infinitesimal space domain \( \Omega \) and a short period of time gives

\[ \int_0^{\theta^*} \int_\Omega \frac{\partial}{\partial t} \eta + \nabla_h \cdot \vec{V} dA dt = \int_0^{\theta^*} \int_\Omega \delta(t)(x-x_s)\delta(y-y_s)dA dt = 1 \]

The second term on the left hand side is negligible if the domain \( \Omega \) is small, and the continuity equation can be simplified to

\[ \left. \int_\Omega \eta dA \right|_{t=0^+} - \left. \int_\Omega \eta dA \right|_{t=0^-} = 1 \]

(2)
That is, the initially elevated volume at \( t=0 \) equals 1. The GF is the response due to impulsive unit volume \( \eta \) increase at the source point \( \mathbf{r}_s = (x_s, y_s) \). This response is denoted as a vector \( G_\eta \):

\[
G_\eta (\mathbf{r}, t; \mathbf{r}_s) = (\hat{\eta}, \hat{P}, \hat{Q})
\]

(3)

where \( \hat{\eta} \) is the \( \eta \) response, \( \hat{P} \) is the response of the variable \( P \), and \( \hat{Q} \) the response of \( Q \) to impulsive \( \eta \) increased.

Thus, the SWEs for a GF can be rewritten as

\[
\hat{\eta} + (\hat{P})_x + (\hat{Q})_y = \delta(t) \delta(x - x_s) \delta(y - y_s)
\]
\[
\hat{P} + gd \hat{\eta} = 0
\]
\[
\hat{Q} + gd \hat{\eta} = 0
\]

(4)

It should be noted that in a discretized numerical simulation, a unit elevation is used as the initial impulse. Hence, the impulsive volume increase for a numerical GF is the area of the source grid, instead of one.

A briefer expression for eq. (4) can be obtained by introducing the operator

\[
O = \begin{bmatrix}
0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\
gd \frac{\partial}{\partial x} & 0 & 0 \\
gd \frac{\partial}{\partial y} & 0 & 0
\end{bmatrix}.
\]

(5)

After expressing the forcing delta-function as a vector with three components corresponding to the three equations of eq. (4),

\[
\delta(\mathbf{r}_s) = (\delta(t) \delta(x - x_s) \delta(y - y_s), 0, 0)
\]

(6)

the SWEs for GF can be simplified to
\( \frac{\partial}{\partial t} G^T = -OG^T + \delta^T \) \hspace{1cm} (7)

Note that the superscript \( T \) represents the transpose of a vector.

### 2.2 SMF Tsunami

If the sea bottom is deformable, the continuity eq. in SWEs should include a new term \(-\partial d/\partial t\), the temporal variation of the sea depth. On the other hand, if the thickness of the slide is much smaller than the total water depth, the contribution of the SMF in the momentum equations is negligible. Thus, the process of tsunami generation can be described by the following matrix equation (Lynett and Liu, 2002; Wang and Liu, 2006):

\[
\frac{\partial}{\partial t} \begin{bmatrix} \eta \\ P \\ Q \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ g d & 0 & 0 \\ g d & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta \\ P \\ Q \end{bmatrix} + \begin{bmatrix} -\frac{\partial d}{\partial t} \\ 0 \\ 0 \end{bmatrix} \hspace{1cm} (8)
\]

Note that the same governing equations are used in the most recent version of the Cornell Multigrid Coupled Tsunami Model (COMCOT; Wang and Power, 2011). With this formulation, tsunami generation is related to the temporal variation of the sea depth. Thus, the SMF continuously contributes to the tsunami. Following the notation of the previous section, we introduce an unknown vector

\[
Z = \{\eta, P, Q\} \hspace{1cm} (9)
\]

and a forcing vector

\[
f = \left\{ -\frac{\partial d}{\partial t}, 0, 0 \right\} \hspace{1cm} (10)
\]

the SWEs governing the generation and propagation of an SMF tsunami then can be expressed in a brief way:
\[
\frac{\partial}{\partial t} Z^T = -OZ^T + f^T.
\]  
(11)

Similar to eq. (7), the superscript T represents the transpose of a vector.

### 2.3 GF and the Quick Forecast of SMF Tsunami

Similarities between the SWEs of GF, eq. (7), and eq.(11) for SMF tsunamis, are obvious. For both equations, the left hand sides are identical. The right hand side for the GF is a delta function, while that for the SMF tsunami is the temporal variation of the sea depth which represents the forcing due to sliding. Applying a Laplace transform, the problem for SMF tsunamis can be solved as the convolution of GF and the forcing as

\[
Z = G * f^T
\]

\[
= \int_0^\infty \int_{\mathbb{D}_h} G(r, \tau; \bar{r}, \bar{r}) \cdot f^T(\bar{r}, t - \tau) d\Omega_d d\tau
\]

\[
= \int_0^\infty \int_{\mathbb{D}_h} \frac{\partial}{\partial t}(\bar{r}, t - \tau) \eta(\bar{r}, \tau, \bar{r}) d\Omega_d d\tau
\]

(12)

Thus, the continuous contribution of the slide can be represented by a convolution.

Besides the convolution, another term will be added if the initial condition is nontrivial. This contribution from the initial sea surface elevation or initial flow are consistent with the elevation GF solution for seismic tsunamis (see, e.g. Chen et al. 2015), which is generated by impulsive fault rupture and can be constructed as a linear combination of GFs. However, if the tsunami calculation starts before the landslide, neither initial flow nor initial elevation exists. Hence, the contribution due to initial conditions vanishes and eq. (12) is the complete solution to the SMF tsunami.

### 2.4 Reciprocity of Green’s Functions

Applying the reciprocity of GF and RGF in the forecast of tsunami was first suggested by Loomis (1979). The first tsunami forecast system that applies the reciprocity of GF and RGF was shown in Chen et al. (2015), which is designed specifically for seismic tsunamis.
Reciprocity between elevation GF and RGF in shallow water equations can be verified numerically. That is, the elevation response to an initial impulsive elevation, and its reciprocal with the locations of source and receiver exchanged, are calculated numerically. The comparison of these two results are shown to be identical, as has been shown in Chen and Liu (2009), Chen et al. (2012) and Chen et al. (2015).

Using RGF instead of GF is done to reduce the computer time in computing the pre-calculated GF: For a large source area there will be many GFs which correspond to the forcing at all source point. Pre-calculation of all the GFs is very time-consuming. Taking the 2011 Tohoku Tsunami for example, the source zone is approximately 500 km long and 200 km wide. A reasonable 2 min. resolution means 10,000 GFs have to be calculated, and the number of GFs increases if more tsunami source locations are to be considered.

For SMF tsunamis, the source zone is not that large. Still the number of possible submarine sliding sites could be more than one and the total number of GFs is large. Thus, using RGF instead of GF is more economical and feasible.

3 Results

In this section, three idealized SMF cases are used to verify the RGF approach. The first two cases are vertical sea bottom movements with different displacement rates. The third case is a historical event following Li et al. (2015), with an idealized truncated hyperbolic slide whose kinematics is described in Enet and Grilli (2007). In each case, direct simulation of COMCOT is compared with an RGF approach and the results agree well with each other. Thus, using RGF with convolution is a fast and accurate substitute for the simulation of SMF tsunamis.

3.1 Fast and Slow Sea Bottom Movements

In the first two cases, simple sea bottom movements are considered. Both the direct COMCOT simulation for the SMF tsunami and the RGF calculation are done over the area 118-121.5° E and 19.8-23.5° N, as shown in Fig.1(a). The spatial resolution is 1 min. and both simulations last 30 minutes. For the first case with fast bottom movement, the area enclosed by the red rectangle to the southwest of Taiwan in Fig. 1(a) is set to move 3 m downward in 120 s. This downward movement occurs uniformly in both space and time. Thus, the source strength due to the bottom subsidence is -0.025 m/s over the whole rectangle within the 120 s extension.

For the calculation of RGF, the initial sea surface elevation of the simulation domain is shown in Fig. 1(b), where the yellow solid square of initial unit elevation is the location of the initial impulse. This location, 120.217° E and 22.583° N with 95 m
water depth, is to represent the incident tsunami at the vulnerable city—Kaohsiung. The evolution of the sea surface over the whole domain following the initial impulse is the desired RGF.

The direct simulation of COMCOT gives the time series of sea surface represented by the red line of Fig. 1(c), and the convolution of the RGF and the constant -0.025 m/s over the red rectangle area from 0s to 120s gives the blue line. The agreement between these two approaches verifies the theory of this study.

Another idealized situation with slow sea bottom change is also considered as Case 2: The area enclosed by the red rectangle to the southwest of Taiwan in Fig. 1(a) is set to move 3 m downward in 600 s. This downward movement is equivalent to a source strength of -0.005 m/s which is uniform in both space and in the 600 s time extension. Comparison between the direct COMCOT simulation and the convolution of the RGF and the constant source strength over the red rectangle area in the 600 s sliding period also gives good agreement, as shown in Fig. 2(b).

### 3.2 Historical SMF Tsunami on the Southwest Coast of Taiwan

For the southwest coast of Taiwan, a tsunami was reported in the year 1781. The record has it that when the fishermen came back after fishing, “they found the houses were submerged and the fishing rafts could sail over the bamboo.” The fishing rafts went out to sea before the tsunami came; therefore, it was a fair day and hence this flooding is due to a tsunami, not a disguised storm surge. Li et al. (2015) called this event a mysterious tsunami because no big earthquakes had been reported, and proposed this devastating tsunami of 1781 to be a SMF tsunami.

Previous studies have shown both the volume and the cross-sectional area of the slide play an important role in tsunami generation (Lo and Liu, 2017). The deformation of the slide does not significantly change the generated tsunami and scenarios generated by a rigid slide body can provide the first order estimate of tsunami wave magnitude (Grilli et al., 2015). Therefore, an idealized model with a rigid slide body is adopted as the third case in the present study.

Following Enet and Grilli (2007), the shape of the sliding zone is assumed to be truncated hyperbolic and the depth is expressed as

\[ z = \frac{T}{1 - \epsilon} \left[ \text{sech}(k_b x) \text{sech}(k_w y) - \epsilon \right], \]

where \( T \) is the thickness,

\[ k_b = \frac{2}{b} \text{acosh} \left( \frac{1}{\epsilon} \right), \]

\[ k_w = \frac{2}{w} \text{acosh} \left( \frac{1}{\epsilon} \right), \]
where \( b \) and \( w \) are the longitudinal and transverse length scales of the slide, and the truncation parameter \( \varepsilon \) is set to be 0.717. Longitudinal and transverse length scales, along with other slide parameters shown in Table 1 have been adopted in Li et al. (2015) and is also used in Case 3 to simulate this historical event in Taiwan. The movement is described by semi-empirical kinematic formulas provided in Enet and Grilli (2007). For example, the slide displacement of the SMF, \( s(t) \), is given as

\[
s(t) = s_0 \ln \left[ \cosh \left( \frac{t}{t_0} \right) \right]
\]

where

\[
s_0 \approx 4.48b
\]

and

\[
t_0 \approx 3.87 \sqrt{\frac{b}{g \sin \theta}}
\]

The displacement \( s(t) \) is explicitly plotted in Fig. 3(a) based on SMF parameters of Table 1. It should be noted that the length scales defined in Table 1 are not exactly the same as Li et al. (2015) because a different slide model is applied in the present study. Our purpose is not to reproduce an SMF tsunami event. Instead, we are testing the RGF approach by using the same information as a direct simulation to verify if the results are comparable.

Similar to the first two cases, the direct COMCOT simulation for the SMF tsunami is done over the area 118-121.5°E and 19.8-23.5°N, with 1 min. spatial resolution and both simulate the sea surface evolution for 30 minutes. The RGF is exactly the same as previous cases and can represent the incident tsunami at Kaohsiung. The water level time series given by direct COMCOT simulation is very close to that of the RGF approach, as shown in Fig. 3(b).

3.3 Computer Time Comparison

Besides accuracy, the efficiency of the RGF method is compared with the direct simulation. For the same desktop PCs with 16GB RAM and Intel i7-7700 CPU, the CPU time for the RGF approach is about 1 s, while a direct COMCOT simulation takes 20 s. As the results of both approaches are identical, the RGF is much more economical than the direct COMCOT simulation.
4 Conclusion and Discussions

RGF approach is economical, fast and robust because the RGF is pre-calculated and no equation-solving is involved. The tsunami waveform can be obtained in 1 s once a submarine landslide is detected. Thus, a tsunami warning can be issued promptly to mitigate possible hazards, with a similar process for a seismic tsunami when an earthquake occurs (see, e.g., Chen et al. 2015).

One problem in the mitigation of SMF tsunami is that the detection technology of SMF is not as mature and comprehensive as that for earthquakes. Earthquakes are serious hazards; advanced technologies have been developed and most earthquake-prone areas have been covered by seismometer networks. Consequently, seismic information usually can be obtained promptly with very high accuracy, while there is usually no access to information on landslides, especially SMFs. However, the quick forecast of SMF tsunamis is still possible. For a detailed simulation of the SMF tsunami, information on the volume, density and cohesive property of the slide material, as well as the location, depth, movement speed, distance and duration of the slide displacement are all needed. Some properties such as density and cohesiveness can be measured beforehand in a survey on coastal seas. Besides, previous studies have shown that both inland and submarine landslides can be detected by hydrophones (e.g., Caplan-Auerbach et al. 2001) or broadband seismometers (e.g., Lin et al. 2010). Thus, it is possible to determine the time and location of the landslide. With idealized models such as Watts (2003) which has been used in this study, as well as information on local bathymetry, the SMF tsunami can be forecast. Available landslide information is much less accurate than the earthquake information used in existing forecast systems for seismic tsunamis. Therefore, a forecast system for SMF tsunami should be more flexible than that of a seismic tsunami so that more situations/scenarios can be included, and a wide range of simulations should be presented. As the RGF approach is economical, fast and robust, more than one simulation result can be given in a reasonable time. Hence it can be concluded that a forecast system can be constructed using RGF: Once a submarine landslide is detected, the range of volume, location, movement speed and other slide information can be estimated. Along with previous knowledge on the local bathymetry and properties of sea bottom sediment, reasonable estimations of the best and the worst (in terms of the devastation induced by the tsunami) situations can be calculated in minutes. Further forecast such as inundation maps can be generated based on the highest tsunami wave height (Chen et al. 2015). Thus, quick forecasting of SMF tsunami is possible and can be used for tsunami hazard mitigation.

Acknowledgements

This research was completed with grants from Ministry of Science and Technology (MOST) of Taiwan, Republic of China (MOST 107-2611-M-110-012 and MOST107-2911-I-110-301).
References


https://en.wikipedia.org/wiki/2018_Sunda_Strait_tsunami

Table 1. The SMF information to the southwest of Taiwan used in third case of tsunami simulation

<p>| | |</p>
<table>
<thead>
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</tr>
<tr>
<td>width</td>
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<tr>
<td>thickness</td>
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</tr>
<tr>
<td>Latitude</td>
<td>22.45°N</td>
</tr>
<tr>
<td>Sliding direction</td>
<td>150°</td>
</tr>
<tr>
<td>Slide duration</td>
<td>0s to 150s</td>
</tr>
</tbody>
</table>
The area moves vertically down 3 meters in 120 seconds.
time: 0day 0hour 0min 0sec

latitude

longitude

sea level (m)
Figure 1: (a) The simulation domain for Cases 1 and 2: 118-121.5° E and 19.8-23.5° N, with the source zone denoted by the red rectangle to the southwest of Taiwan. (b) The initial sea surface elevation for the calculation of RGF. The location of the initial impulse, 120.217° E and 22.583° N, is used to represent tsunami near Kaohsiung. (c) The simulated sea surface time series by direct COMCOT simulation (red) and the RGF approach (blue).

Figure 2. Comparison between the direct COMCOT simulation (red) and the RGF approach (blue) of Case 2.
Figure 3. (a) Movement of the 1781 SMF described by semi-empirical kinematic formulas of Enet and Grilli (2007). (b) Comparison between the direct COMCOT simulation (red) and the RGF approach (blue) of Case 3.