Dear Thierry,

**OUR MASS BALANCE is RIGHT!**

We apply the most elementary relation of hydraulics to describe the pile-up process, simply the flux of incoming mass into a bucket is equal to the speed the surface is rising in the bucket (see picture below). You will have us believe that the speed the level in the bucket is rising is faster because a “shockwave” propagates against the incoming flow. You argue that because the two are moving against each other, the incoming mass flux is larger and therefore the growth of the piled-up mass increases faster than we feed it!

What you forget is simple: in our case the “shockwave” is the mass flux. In the theory of shockwaves, the speed of the shock is due to the exchange of kinetic and potential energy (elasticity). It therefore has another source. When the mass balance is the source of the “shock”, then the “shock” cannot modify the source. Your speed, $u_n$, must therefore be independent of the mass flux. When this is so, your mass balance model is correct (for example, in the case when $u_n$ is equal to the speed of sound). In our case, we have no shock – we simply have mass piling-up in a “bucket” and consider the speed the surface is rising. This is again where the analogy with “shockwaves” breaks down completely.

Frankly, the rise of the surface level in the bucket is independent of whether we sit on the surface or consider it from the observer at rest (simple Galilean transformation). Sitting on the interface, I see the wall moving backwards, moving with the speed of $Sdot$ (or $u_n$) and therefore it seems as the particles would flow with the speed $Sdot$ through interface, such that for the outside observer the mass stops. In your model you only acquire a surplus of mass (incoming $Sdot h1 \rho_1$) and forget the same amount is going into the pile-up region with the speed $Sdot$. Mathematically, your model of mass balance is equivalent to filling the bucket with a hose where the hose is submerged in the
bucket (-u_n A1). You always subtract this term (u_n A1 rho1) or (u_n h1 rho1) from the correct mass balance but never add it back. This produces pile-up speeds that are too high. This is clearly not the pile-up process as we observe it. Furthermore, the question arises where is the interface?

To be honest, the analogy to shockwaves cannot be used at all to describe the pile-up process of an inelastic material, what avalanche snow is. We don’t wish to be impolite, but we simply cannot understand why something so simple as filling a bucket with mass should be treated so complicated. Again, the application of the work energy theorem and simply hydraulic mass balances are sufficient to describe the external pile-up forces on structures.