Responses to RC3

October 3rd, 2018

Re: Re-submission of manuscript “Development of a methodological framework for the assessment of seismic induced tsunami hazard through uncertainty quantification: application to the Azores-Gibraltar Fracture” by Vito Bacchi et al. NHESS-2018-142.

Dear RC3,

We appreciate your careful review and constructive suggestions. It is our belief that the manuscript is substantially improved after making the suggested edits. Following this letter are the reviewer comments with our responses in italics, including how and where the text was modified. Changes made in the manuscript are marked in blue (see attached file).

According to the comments from Dr. Luis Matias (RC1), we propose to modify the title of the paper as follow:

“Development of a methodological framework for the assessment of seismic induced tsunami hazard through uncertainty quantification: the test case of the Azores-Gibraltar Plate Boundary”.

Thank you for your consideration.

Sincerely,
Vito Bacchi on behalf of authors

Major comments:

(1) The manuscript lacks some important references on one of the main topic of the manuscript itself: quantification of uncertainties in tsunami hazard assessment. Several works faced this topic (among others, Sorensen et al, 2012, Horspool et al, 2014, Davies et al., 2016, Lorito et al., 2015, Selva et al., 2016) using probabilistic approaches and proposing methods to strongly reduce computation costs. Also approaches to develop databases based on combination of elementary sources to speed up tsunami modeling have been proposed (Miranda et al, 2015 and Baptista et al, 2017), even with the quantification of the associated uncertainty (Molinary et al., 2016). I think that the manuscript should be rethought considering the existing framework.

The authors agree with RC3 and consider that the first version of the paper lacks these important references. From a “bibliographic” point of view, the first version of the paper was incomplete. In our study we focused on deterministic tsunami hazard assessment (DTHA) and not on the probabilistic approaches. This key aspect of the paper was probably not clearly explained in the text and we strongly modified the structure of the revised paper in order to better define the objectives and the methodology we proposed. Furthermore, in order to complete the “state of the art” concerning uncertainties, we introduced the mentioned work in the revised paper (mainly in the section 2.3).

(2) I am a bit lost between the explanation of the method and how it was applied for the case study. In particular, the environment Promethee seems to me the glue of everything, so I found strange that it was quickly mentioned only in section 3. I would suggest to present what Promethee is, which module contains, how is used in this work and what the authors customized in the frame of Promethee for their scopes. For example, I did not understand subsection 2.2: why using a subsection to describe a module that was not used (if I understand well), i.e., the Monte Carlo method originally implemented in Promethee? Authors could state that uncertainty propagation used in Promethee environment for this
study is modeled using an external package based on kriging. My suggestion is to rearrange section 2 and 3 in a clearer way.

We agree with RC3: the structure of the paper was definitely not clear. As a consequence, the organization of the paper was changed and the methodology more detailed. A flow diagram of the main three steps of the methodology is reported on the new Figure 1 (the ancient was removed). Moreover, the section 2 is reorganized in order to introduce the proposed steps for the application of the methodology and provide further details. In the revised paper, we first introduce the main three steps of the proposed methodology (beginning of the section 2), and then we detail each step. Promethee is introduced at the beginning of the section 2 (section 2.1).

(3) Even though I understand that the aim of the paper is illustrative of the method, I have some doubts about the design database: the number of considered scenarios (about 5,000) seems to me relatively low with respect the number of the considered parameters and their large variability. My concern is that a too rough sampling of input parameters could introduce bias in the design of the meta-model. I would ask to authors to address this issue.

We chose to perform a fairly limited number of tsunami scenarios to cover a large range of tsunami input parameters (9, as reported in the text). However, we propose two kinds of validations of our meta-models: (i) the analysis of the R2 (eq. 6) and (ii) of residuals (eq. 7) on the testing data-base. This kind of analysis allows us to evaluate the uncertainty related to the use of a meta-model instead of the original model (as reported in section 2.2.4 of the revised manuscript).

To better address this issue, we complete the section 3.2.1 of the revised paper with the sentence reported below:

“If the validation results are not satisfactory, this would suggest that other simulations need to be performed to complete the design data-base and reduce the excessive bias derived from a rough sampling of the input parameters.”

(4) The work often refers to the meeting communication by Antoshchenkova et al., (2016) which cannot be verified by readers. This should be avoided in a peer-reviewed manuscript. When needed, important element from that work should be reported directly here.

We agree with RC3. The citations to Antoshchenkova et al., 2016 and Imbert et al., 2015 were delated as these references are not available in open-access literature. Instead, we present only the hypothesis employed for the construction of the design data-base, which is based on Monte-Carlo sampling of the inputs parameters reported in Table 2 (of the revised paper) and also introduce the numerical domain (Figure 2 of the revised paper) and the numerical model used for the simulations of the design data-base (sec 2.1 of the revised paper). Moreover, for more details, we reported in the revised paper that all the equations used for the modelling of tsunami generation and propagation are reported in Allgeyer et al. (2013).

(5) About tsunami modeling: linear or non-linear shallow water equations have been used? This is not specified in the text, however depth of tide gauges is 40 m or less (and less than 30 m for 3 stations), where non linearity effects become very strong and cannot be neglected.

In the revised version of the paper we specify that the equations of tsunami modelling are fully reported in Allgeyer et al. (2013), by introducing this sentence (section 2.1 of the revised paper):

“Then, the computation of the tsunami propagation is based on hydrodynamic equations, under the nonlinear shallow water approximation (the Boussinesq equations as reported in Allgeyer et al., 2013).”
Concerning the roughly approximation related to the bathymetrical greed used in this study, we also specify in the text that (section 2.1 of the revised manuscript):

“In this study, all the simulations were performed on the same bathymetric grid with a space resolution of 2’ (~3.6 km). The numerical model was not directly validated by the comparison with similar simulations from literature. Considering the rough bathymetrical grid resolution, the developed numerical model is not adapted to the estimation of the tsunamis run-up and the inundation areas and it can’t be used for a real assessment of the tsunami hazard along the French Atlantic Coast. However, this work being methodological, the authors consider that the numerical results are consistent with the objectives of the study. Moreover, the tsunami-code was largely validated through extensive benchmarks in the framework of the work package 1 of the TANDEM research project (Violeau et al., 2016) by ensuring its ability to reproduce tsunamis generation and propagation. As a consequence, the order of magnitude of the tsunami height computed in this study should be realistic and adapted to the test of the methodology.”

Finally, we consider that the non-linear Boussinesq equation is adapted to the simulation of tsunami propagation near to the coast-line and also for the simulation of tsunami run-up and propagation in the inundated areas, as showed in literature (i.e Poisson et al., 2009; Allgeyer et al., 2013). Moreover, in the framework of the French research project TANDEM dedicated to tsunami modelling, a series of benchmarks has been set up, addressing the various stages of a tsunami event: generation, propagation, run-up and inundation (Kazolea et al., 2017). The authors present the results of five codes, involving both depth-averaged Boussinesq and fully 3D Navier-Stokes equations, aimed at being applicable to tsunami modelling. According the authors, the five codes can be classified in two main categories: those based on the Navier-Stokes (NS) model (Thesis and EOLE) and the depth-averaged Boussinesq-type (BT) models (TUCwave, SLOWs, FUNWAVE-TVD). The authors conclude that both categories provide very similar and satisfactory results on the whole (according to experiment) test-case.

(6) I did not understand if magnitude is uniformly sampled as stated in the text (page 14, line 17), since in figure 9a the distribution seems very different. Authors should clarify this point.

We insert in the section 3.1 (Design data-base for meta-models construction) of the revised paper the following sentence to clarify this issue:

“In order to build the design database, fault parameters as defined in section 2.1 and Table 2 were sampled randomly and independently with Monte-Carlo method and supposing uniform distributions. The uniform distribution was chosen in order to build meta-models able to reproduce tsunamis heights generated by various seismic sources with the same accuracy. The resulting earthquake magnitudes are computed using the sampled parameters with equation 2. The shear modulus chosen for the magnitude estimation is a constant value assumed to be equal to 30 GPA.”

(7) Even though the manuscript is illustrative, maybe to make the case study less far from something more realistic, the authors could use a magnitude distribution from catalogs.

The objective of our work, which was not clearly defined in the original paper, is to propose a new methodology for the improvement of DTHA. In the revised paper, we better detail the objectives of our paper and the differences with the classical deterministic approach based on the definition of a Maximum Credible Scenario (MCS):

“Nowadays, in the classical DTHA, as reported in the cited papers, MCS is mainly focused on seismic source parameters (MCS_p) and not on the tsunamis heights at a given location (MCS_h). The great M 9.0 Tohoku-Oki subduction earthquake of 2011, the largest ever recorded in Japan (Saito et al., 2011), has clearly shown the limitations of the MCS_p approach. In fact, considering the uncertainties related to the characterization of seismic sources, using MCS_p for DTHA may lead to an underestimation of MCS_h in the area of interest.”
In this context, the objective of our work is to propose a new methodology to go beyond the MCS_p approach and focus more on MCS_h through the evaluation of all the possible tsunami heights at a given location. The MCS_h approach has the advantage that it accounts for other scenarios, generated by potential seismic sources not associated to the MCS_p but generating tsunami heights potentially impacting the zone to protect. The proposed methodological framework for the improvement of DTHA is based on classical uncertainty quantification (UQ) techniques (i.e. Saltelli et al., 2000; Saltelli et al., 2008), which are robust statistical tools permitting to largely explore a given seismic area and provide all the possible tsunami heights generated by this area. This approach is a new philosophy for DTHA and can also permit to ensure that MCS_p is the MCS_h scenario for the target area.”

(8) The numbering of figures and tables should be adjusted with respect the order in which they are mentioned in the text.

These modifications were integrated to the revised version of the manuscript.

Other comments:

(1) Page 2, lines 28-30: If I understand well the sentence, I disagree with the authors, since I would say the opposite: if the knowledge on the area is poor, a probabilistic approach can at least quantify this level of uncertainty, whereas deterministic approaches hardly can catch what is unknown and they could suffer of relevant bias in the hazard analysis.

We agree that this sentence is ambiguous and not really justified when considering the objectives of our work. remove the sentence.

(2) Page 3, lines 13: The authors are referring to deterministic approaches only or to probabilistic ones as well?

We only refer to DTHA. We modify the paper in order to better distinguish DTHA from PTHA.

(3) Page 17, line 17: I cannot really understand the meaning of the sentence “for a given magnitude, a tsunami generated by a well located-oriented fault would be potentially more hazardous”.

In the revised version of the paper we clarify the arguments that lead us to this sentence. Especially, in section 4 of the revised paper we analyse MCS_h. This analysis is mainly focused on three points:

- the choice of the hazard level and the analysis of the strongest scenarios of MCS_h (above the proposed hazard level);
- the comparison with MCS_p results
- the sensibility analysis using Jansen’s method

The analysis of the strongest scenarios (stronger water height at a given location of the French Atlantic Coast) suggests that a part of the strongest tsunami height are generated by “oceanic shelf” seismic sources with relatively “low” magnitudes (between 7.8 and 8.2). From this first result we suggest that the location of seismic sources can be relevant for MCS_h along the French Atlantic Coast.

Then, the authors focused on sensibility analysis using the Jansen’s method which permits to explore the space of the input parameters in order to assess their influence on model results. This sensibility is estimated through the Sobol indices which are a measure of how the variance of each parameter influence the global variance of the variable of interest (in this study, the maximum tsunami water height). Concretely, for the evaluation of Sobol indices through the Jansen’s method, other evaluation of the meta-model (and thus, other simulations) are evaluated. The space of input parameters is
sampled in an adapted way in order to compute correctly the Sobol’s indices and the convergence is checked (we verify that the number of evaluation used for the sobol indices computation is sufficient to obtain a convergent value of the Sobol index). This is managed by the R-package sensitivity cited in the paper. From these results, we conclude that a “well located-oriented fault” can be relevant for the North Atlantic Gauges, even if the slip is still the more relevant parameter. Moreover, the author underline that these results are consistent with previous results from Allgeyer et al. (2013), indicating that some areas along the French Atlantic Coast are more exposed to tsunamis from AGFZ.

(4) Figure 3: I guess that tick labels for longitude axis should be W instead O.

Figure 3 was modified.

Bibliography:


Development of a methodological framework for the assessment of seismic induced tsunami hazard through uncertainty quantification: the test case of the Azores-Gibraltar Plate Boundary

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Abstract. The aim of this study is to propose a new methodology for deterministic tsunami hazard assessment based on the analysis of all the possible tsunami heights at a given location. With this new approach, the hazard level is evaluated through a numerical database constructed using “uncertainty quantification (UQ) techniques” which allows the exploration of the tsunamigenic potential of a seismic zone in a rigorous and complete way. This concept goes beyond the definition of the Maximum Credible Scenario based on the definition of particular source parameters and classically reported in the literature. The proposed methodology relies on the construction and validation of “emulators”, or “meta-models”, that drastically reduce the computational time necessary for tsunami simulations. It is the first time, to our knowledge, that meta-models are used in this way. To test the methodology, a numerical database of nearly 50,000 tsunami scenarios generated by the Azores-Gibraltar Plate Boundary (AGPB) and that may potentially impact the French Atlantic Coast was constructed. Tsunami heights distributions resulting from the UQ using meta-models are then discussed to illustrate the advantages of such an approach in decision-making compared to results which can be obtained with a more classical scenario-based approach. In particular, the results suggest that the most influential parameters controlling tsunami heights generated in the AGPB is not only the magnitude but also the location of the impacted zone. It must be underlined that results from this study are used to illustrate the general methodology through a case study with simplified hypothesis and should not be considered for an operational assessment of tsunami hazard along the Atlantic Coast.

1 Introduction

Tsunami hazard science developed intensively about 50 years ago (Zoback et al., 2013) following four complementary approaches: the (i) identification of past tsunamis in the geologic record on land, the (ii) characterization of tsunamis of different source types, the (iii) hydrodynamic modelling to predict run-up and inundation areas and the (iv) ocean buoy systems for tsunamis detection and recording. Even if it is generally accepted that large-scale tsunamis can be
generated by three main types of geologic events, namely the earthquakes, the submarine and subaerial landslides, and a variety of mechanisms associated with volcanism, in this study we only focused on tsunamis generated by earthquakes.

Earthquakes are the most common source of large-scale tsunamis, with dip-slip earthquakes (i.e. with vertical movement) being more tsunamigenic than strike-slip earthquakes (i.e. with horizontal movement). It is generally observed that only large magnitude earthquakes (M>6.5) lead to widely observable tsunamis, because earthquakes need to be large enough to be able to produce significant displacements at the sea bottom surface (Wells and Coppersmith, 1994). The typical tectonic environment able to generate tsunamigenic earthquakes is the subduction zone where major tsunami earthquakes occur on shallow inter-plate thrusts. However, they may also be generated by outer rise earthquakes, either within the subducting slab or the overlying crust (Satake and Tanioka, 1999). Other oceanic converging boundaries may also be responsible for tsunamis even if not classified as subduction zones, as the North-African coasts for instance Álvarez-Gómez et al. (2011). On the opposite, diverging plate boundaries are also able to generate tsunamis although earthquakes are in general lower in magnitudes in comparison to subduction zones (Ambraseys and Synolakis, 2010). Finally, purely strike-slip earthquakes may also affect oceanic floors, but they are in general not able to generate major tsunamis, unless a significant earthquake vertical component exists or if steep slopes are affected (Tanioka and Satake, 1996).

Two approaches are widely used by the scientific community to quantify tsunami hazard assessment: the scenario-based (or deterministic - DTHA) and the probabilistic approaches (Omira et al., 2016). The probabilistic approach (PTHA) relies on four basic steps (Geist and Lynett, 2014): (i) the specification of source parameters, including source probabilities for all relevant tsunami sources, (ii) the choice of a probability model that describes source occurrence in time (most often Poisson), (iii) the hydrodynamic modelling for each source location and set of source parameters to compute tsunami hazard characteristics at a site (or at a point) of interest and (iv) the aggregation of the modelling results and incorporation of uncertainty. PTHA calculates the likelihood of tsunami impact using a large number of possible tsunami sources and including the contribution of small and large events from both near- and far-field (Omira et al., 2016). The objectives of PTHA are to condense the complexity and the variability of tsunamis into a manageable set of parameters and to provide a synopsis of the tsunami hazard along entire coastlines in order to help identifying vulnerable locations along the coast and specific tsunami source regions to which these vulnerable locations on the coastline are sensitive (Geist and Parsons, 2006). Early studies are based on different assumptions (i.e. Rikitake and Aida, 1998; Downes and Stirling, 2001) and a surge of PTHA studies started in the early 2000s up to present (i.e. Geist and Parsons, 2006; Annaka et al., 2007; Geist et al., 2009; González et al., 2009; Blaser et al., 2012), in particular after the major tsunamis in Indonesia in 2004 and in Japan in 2011.

In seismic areas with no (or poor) knowledge of the faults and crustal characteristics it is not trivial to associate a probability distribution to the seismic inputs parameters and DTHA may be more adapted than the probabilistic method for the assessment of tsunami hazard. A scenario-based (or DTHA) approach classically relies on the study of “maximum credible scenarios” (MCS). The tsunami hazard is assessed by means of considering particular source scenarios (the MCS with respect to the actual knowledge of the tsunamigenic seismic sources) and then estimating the coastal tsunami impact through numerical modelling (JSCE, 2002; Lynett et al., 2016), without addressing the likelihood of occurrence of such a big
event (Omira et al., 2016). The outputs of the deterministic analysis are, in general, tsunami travel time, wave height, flow depth, run-up, and current velocity maps corresponding to the chosen scenario (Omira et al., 2016). However, DTHA could be hampered by the use of specific values of input parameters which may be subjective depending on the person or group carrying out the analysis (Roshan et al., 2016). A good example is the 1755 Lisbon tsunami, generated by an earthquake in the Azores Gibraltar Plate Boundary (AGPB). The Great 1755 Lisbon earthquake generated the most historically destructive tsunami near the Portugal coasts (Santos and Koshimura, 2015). Source location and contemporary effects of such tsunami are not precisely identified and several earthquake scenarios have already been published in the literature since last decades (Baptista et al., 1998; Grandin et al., 2007; Gutscher et al., 2006; Horsburgh et al., 2008; Johnston, 1996; Zitellini et al., 1999; Baptista et al., 2003; Terrinha et al., 2003; Gracia et al., 2003; Barkan et al., 2009; Cunha et al., 2010). All of these studies, mainly based on a comparison of several fault hypotheses based on geological properties of the AGPB, show how variable parameters of the seismic source can be, depending on the studied fault and focal mechanisms.

Nowadays, in the classical DTHA, as reported in the cited papers, MCS is mainly focused on seismic source parameters (MCS_p) and not on the tsunamis heights at a given location (MCS_h). The great M 9.0 Tohoku-Oki subduction earthquake of 2011, the largest ever recorded in Japan (Saito et al., 2011), has clearly shown the limitations of the MCS_p approach. In fact, considering the uncertainties related to the characterization of seismic sources, using MCS_p for DTHA may lead to an underestimation of MCS_h in the area of interest. In this context, the objective of our work is to propose a new methodology to go beyond the MCS_p approach and focus more on MCS_h through the evaluation of all the possible tsunamis heights at a given location. The MCS_h approach has the advantage that it accounts for other scenarios, generated by potential seismic sources not associated to the MCS_p but generating tsunamis heights potentially impacting the zone to protect. The proposed methodological framework for the improvement of DTHA is based on classical uncertainty quantification (UQ) techniques (i.e. Saltelli et al., 2000; Saltelli et al., 2008), which are robust statistical tools permitting to largely explore a given seismic area and provide all the possible tsunami heights generated by this area. This approach is a new philosophy for DTHA and can also permit to ensure that MCS_p is the MCS_h scenario for the target area.

In fact, despite the natural complexity of seismic zones prone to tsunamis, in DTHA studies the uncertainties related to model parameters are generally not taken into account in a rigorous and robust way (Lynett et al., 2016; JSCE, 2002), considering the mathematical framework available in the literature (Saltelli et al., 2004; Saltelli et al., 2008; Faivre et al., 2013; Iooss and Lemaître, 2015), and the classical approach to deal with uncertainties consists to perform a limited number of deterministic simulations with penalized values of the seismic sources. For instance, Allgeyer et al. (Allgeyer et al., 2013) modelled the impact of three different scenarios for the 1755 earthquake on the French Atlantic Coast, with a focus on La Rochelle harbour. The authors show that, depending on the source hypothesis and tide conditions, several areas (western part of the island of Re´ and northern coast of the island of Oléron) may have experienced a moderate impact from 0.5 to 1 m for this tsunami event. Recently, Roshan et al. (2016) tried to improve the DTHA procedure detailed in Yanagisawa et al. (2007) in order to better evaluate the effects of the seismic source uncertainties. The authors focused on a possible range of parameters which could produce more robust estimates of hazard instead of using single value of each seismic source
parameter (Roshan et al., 2016). Basically, their methodology is based on the classical steps of an uncertainty propagation study: (i) the identification and characterization of tsunamigenic earthquake sources, (ii) the estimation of rupture parameters and associated parametric variations, (iii) the exploration of a tsunami numerical model and, finally, (iv) the parametric study capturing the uncertainties through the numerical simulations of the tsunami waves associated to different model inputs parameters varying within a given range. The deterministic numerical results are then processed and hazard maps of mean simulated water height are produced to assess the tsunami hazard (Roshan et al., 2016). Finally, the mean simulated water height of the different tsunami scenarios is employed to design flood level due to tsunami along Indian coast (Roshan et al., 2016). The objective of this exploratory and innovating methodological work was not directly the analysis of the benefits of uncertainty quantification on DTHA (Roshan et al., 2016), even if the authors presented an improvement of the classical MCS approach by considering the uncertainties related to a limited number of seismic source parameters (the dip angle, the strike and the source location), leading to a limited number of tsunami scenarios (around 320). In this sense, they were not interested, for instance, to estimate the robustness of the mean simulated tsunami height considering the low number of simulations. Moreover, the sensitivity of model results to the input parameters was not studied (Roshan et al., 2016). Finally, the other seismic source parameters (e.g. the mean rupture length and width, the slip and the magnitude) were considered as constant for each tsunami scenario and their uncertainty was not evaluated, thus strongly limiting the impact of their study with respect to a more classical uncertainty quantification approach.

In this study, because of the computational constraints associated with UQ, the proposed methodology relies mainly on the use of a meta-model, an emulator of the tsunami numerical model, that make it possible to perform a large number of tsunami scenarios with reduced computational time and, consequently, to intensively explore a tsunamigenic area for which geological and geophysical datasets are in general limited. Statistical emulators have already been employed in the field of tsunamis, for the analysis of landslide-generated tsunamis (Sarri et al., 2012) or for the evaluation of the uncertainty in the friction parameterization used in tsunamis simulations (Sraj et al., 2014). More recently, Rohmer et al. (2018) proposed a Bayesian procedure to infer (i.e. learn) the probability distribution of the source parameters of the earthquake, based on the combination of a kriging-based metamodeling technique, to overcome the high computation time cost of the numerical simulator, and an Approximate Bayesian Computation (ABC) procedure, to perform the Bayesian inference. The authors applied the procedure to the Ligurian (North West of Italy) 1887 tsunami case. All the mentioned studies show that it is of interest to use meta-models for tsunamis analysis in many contexts and at least for reducing the computational time necessary for tsunami simulations (Sarri et al., 2012).

In this research work, the constructed meta-models are exploited with the statistical criteria classically employed in UQ studies (Saltelli et al., 2000; Saltelli, 2002; Saltelli et al., 2008; Iooss and Lemaître, 2015) for the assessment of DTHA. To our knowledge, it is the first time that meta-models are tested for the assessment of DTHA. The methodology was hereafter applied to the AGPB area by the construction and the validation of four meta-models able to reproduce the maximum tsunami heights along the French Atlantic Coast. The AGPB area was chosen as an exercise because of the large available literature, because it represents one of the most important sources of potential earthquake-related tsunamis for the French...
Atlantic coast (as attested by the impact of the 1755 tsunami along the French coast, http://tsunamis.brgm.fr/) and because the results of our constructed meta-models could be compared with previous deterministic simulations of tsunamis from AGPB along the French Atlantic Coast (Allgeyer et al., 2013).

Finally, in section 4 we present how the numerical data-base constructed through UQ techniques should be employed for DTHA along the French Atlantic Coast and its critical analysis. The analysis is focused on the hazard-level which should be retained from UQ (MCS_h) and on the comparison between the tsunami data-base with tsunamis heights obtained with a more classical approach (MCS_p). It must be noted that the aim of this study is to show how a numerical database constructed using UQ “techniques” can be a useful tool for the analysis of the tsunamigenic potential of a seismic zone. As a consequence, this paper illustrates a methodology through a case study, and is not to be considered as an operational assessment of tsunami hazard along the French Atlantic Coast.

2 Presentation of the methodology: theoretical background

In this study, we propose a general methodological framework for the exploration of the tsunamigenic potential of a given seismic area and the quantification of the associated uncertainties. This methodology should permit to identify all the possible tsunamis heights at a given location through UQ and then to assess a hazard level (MCS_h). It should be suitable in two situations: (i) for poorly known tsunamigenic areas with no (or poor) information related to seismic sources or (ii) to evaluate the robustness of the MCS_p, by ensuring that this scenario is really the more impacting for the area to protect. The proposed approach relies on three main steps, as reported in Figure 1. STEP 1 consists in the construction of a numerical model able to reproduce the tsunami heights generated by a given seismic area and impacting a target area. In STEP 2, the numerical model simulates a regular set of physical tsunamigenic scenarios (so called design data-base) that are used for the construction and the validation of an emulator (the meta-model) able to reproduce the original model results in the target zone. In STEP 3, the validated meta-models are used for DTHA. The UQ performed using meta-models instead of the original model permits to explore intensively the tsunamigenic area with nearly zero computational time. The distribution of tsunami heights resulting from UQ (step 3.1) can then be analysed and discussed in order to define a given hazard level for the zone to protect (CMS_h), as presented in section 4. Finally, in order to identify the model parameters contributing the most to the tsunamis height at a given location a global sensitivity analysis (step 3.2) is performed. The mathematical hypothesis of each step are described in the next paragraphs, with a focus on the AGPB. This methodology is general and can be used in different hydraulic fields.

2.1 Step 1: numerical tool for tsunami simulations

The first step consists in the construction of a tsunami numerical model of the area to explore. In this study, the tsunami numerical simulations were performed by using the CEA tsunami code which exploits two models, one for tsunami initialization and the other one for tsunami propagation. The initial seabed deformation caused by an earthquake is generated
with the Okada model (Okada, 1985) and is transmitted instantaneously to the surface of the water. This analytical model satisfies the expression of the seismic moment $M_o$:

$$M_o = \mu \cdot D \cdot L \cdot W$$  \hspace{1cm} (1)

where $\mu$ denotes the shear modulus, $D$ [m] the average slip along the rupture of length $L$ [m] and width $W$ [m]. Then, the seismic magnitude “$M_w$” is directly computed through equation 2, as follow:

$$M_w = \frac{2}{3} \cdot \log_{10}(M_o) - 6.07$$  \hspace{1cm} (2)

The following parameters are also required for tsunami initiation: longitude, latitude, and depth [km] of the center of the source, strike [degrees], dip [degrees], and rake [degrees]. A conceptual scheme of the input parameters for the tsunami-code and the numerical domain used in this study are reported in Figure 2. Then, the computation of the tsunami propagation is based on hydrodynamic equations, under the nonlinear shallow water approximation (the Boussinesq equations as reported in Allgeyer et al., 2013). Shallow water equations are discretized using a finite-differences method in space and time (FDTD). Pressure and velocity fields are evaluated on uniform separate grids according to Arakawa’s C-grid (Arakawa, 1972). Partial derivatives are approximated using upwind finite-differences (Mader, 2004). Time integration is performed using the iterated Crank-Nicholson scheme. No viscosity terms are taken into account in our simulations. The only parameters of this second model are the bathymetry (space step and depth resolution) and the time step.

In this study, all the simulations were performed on the same bathymetric grid with a space resolution of 2’ (~3.6 km). The numerical model was not directly validated by the comparison with similar simulations from literature. Considering the rough bathymetrical grid resolution, the developed numerical model is not adapted to the estimation of the tsunamis run-up and the inundation areas and it can’t be used for a real assessment of the tsunami hazard along the French Atlantic Coast. However, this work being methodological, the authors consider that the numerical results are consistent with the objectives of the study. Moreover, the tsunami-code was largely validated through extensive benchmarks in the framework of the work package 1 of the TANDEM research project (Violeau et al., 2016) by ensuring its ability to reproduce tsunamis generation and propagation. As a consequence, the order of magnitude of the tsunami heights computed in this study should be realistic and adapted to the test of the methodology.

Finally, in order to perform the numerical simulations needed for the meta-model construction and validation (see section 2.2), the CEA code was coupled with the IRSN Promethee bench. Promethee is an environment for parametric computation that allows carrying out UQ studies, when coupled (or warped) to a code. This software is freely distributed by IRSN (http://promethee.irsn.org/doku.php) and allows the parameterization with any numerical code and is optimized for intensive computing. Promethee was first linked to the numerical code by means of a set of software links (similar to bash scripts). In this way, numerical simulations were directly lunched by the IRSN environment. Then, the statistical analysis, such as the Monte-Carlo simulations used for the meta-model construction (see section 2.2.1) and UQ (see section 2.3.1) or
the Sobol indices computation (see section 2.3.2) were also driven by Promethee, which integrates R statistical computing environment by permitting this kind of analysis (R Core Team, 2016).

2.2 Step 2: Meta-model design and validation

For the evaluation of numerical codes in industrial applications, where the computational code is expensive in computation time, so that it cannot be evaluated intensively (e.g. only several hundred calculations are possible), it is usually not easy to estimate some numerical parameters (i.e. the sensitivity Sobol indices), requiring for each input several hundreds or thousands of evaluations of the model (Iooss and Lemaître, 2015). As a consequence, a meta-model instead of the model is generally used in the estimation procedure. A meta-model is a mathematical model of approximation of the numerical model, built on a learning basis (Fang et al., 2006). The meta-model solution is a current engineering practice also for estimating sensitivity indices (Gratiet et al., 2016). In this study, we use the Gaussian process meta-model (also called kriging) (Sacks et al., 1989), which good predictive capacities have been demonstrated in many practical situations (see Marrel et al., 2008, for example). As a consequence, kriging meta-model will be useful both for sensitivity analysis and for a numerical database construction through uncertainty quantification (as reported in the next section). The classical steps for meta-models construction and validation are reported in various studies (i.e. Faivre et al., 2013; Saltelli, 2002; Saltelli et al., 2008), and can be shortly summarized as follow (see Table 1).

The mathematical construction of the meta-model obviously depends on the chosen meta-model. The theoretical background for kriging meta-model design used in this study is fully detailed in Roustant et al. (2012). The associated equations are implemented in the R-packages DiceKring and DiceOptim and reported in the APPENDIX 1 for the interested reader.

2.2.1 The initial design for meta-model fitting

For computer experiments, selecting a design database (or space filling design) is a key issue in building an efficient and informative meta-model (Iooss et al., 2010). The design of the experience is the choice of the input parameters (and, consequently, of the initial simulations) to use in order to build the most accurate meta-model at a minimum computational coast (minimum number of model simulations). It can depend on the chosen meta-model (Kleijnen, 2005). For instance, for Gaussian emulators as kriging, one of the most popular design type is the Latin Hypercube Sampling (LHS) (McKay et al., 1979), which offers flexible design size for any number of simulation inputs (Kleijnen, 2005). An equivalent approach, used in this paper, is the Monte-Carlo sampling technique which also permits a regular and uniform exploration of the space of the model input parameters. In fact, except for some numerical tests reported in previous studies (Iooss et al., 2010), at the moment, no theoretical results gives the type of initial design leading to the best fitted meta-model in terms of meta-model predictions.
2.2.2 The partitioning of the original data base

It is generally assumed that the simulations performed to build the meta-model (the design database) can be partitioned in a training set (data used for the estimation of the meta-model) and a validation set (simulations performed to test the ability of the meta-model to reproduce the model results with other data). According to Amari et al. (1997), to obtain an optimum unbiased meta-model, nearly 80% of the simulations of the design database must be used for the training set and 20% for the validation set. This ratio is used in our study.

2.2.3 The validation of the chosen type of meta-model: training set

In order to assess if a given meta-model (i.e. kriging, gam method, random forest, chaos polynomials, and so on…) is adapted to reproduce the model behaviour for the design database, probably the simplest and most widely used method is the cross-validation or K-fold cross-validation method (Hastie et al., 2002). This step is performed with the training set. The principle of cross-validation is to split the data into K folds of approximately equal size $A_1, A_2, \ldots, A_K$. For $k = 1$ to $K$, a model $\hat{Y}^{(-k)}$ is fitted from the data $U_{j \neq k} A_k$ (all the data except the $A_k$ fold) and this model is validated on the fold $A_k$. Given a criterion of quality $L$ as the Mean Square Error (Eq. 3):

$$L = MSE = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2$$

The quantity used for the “evaluation” of the model is computed as follow:

$$L_k = \frac{1}{n/K} \sum_{i \in A_k} L(y_i, Y^{(-k)}(x_i))$$

Where $\hat{y}_i$ and $y_i$ are, respectively, the meta-model and the model response and $n$ is the number of simulations in the $k$th sample. Finally, the cross-validation used in this study is evaluated through the mean of the quantity $L_k$ computed for each fold:

$$L_{-CV} = \frac{1}{K} \sum_{k=1}^{K} L_k$$

It must be noted that when $K$ is equal to the number of simulations of the training set, the cross-validation method corresponds to the leave-one-out technique (Oliveira, 2008). The methodology employed is described in the DiceEval R-package reference-manual (Dupuy et al., 2015). We considered $K = 10$ and compute the MSE between meta-model approximation and the k-fold observations for each fold ($k$ times). The mean MSE is an index of the mean differences between meta-model predictions and model results, thus representing the ability of the chosen meta-model to reproduce the original model. The more this index approaches zero, the more the meta-model can be considered accurate.
### 2.2.4 Quantification of the meta-model uncertainty: residuals analysis

This last operation is performed on the validation-set (composed of the 20% of the design database). The objectives are (i) to evaluate the meta-models built on the entire testing set (after cross-validation) outside their construction domain (training set) and (ii) to estimate the uncertainty related to the use of the meta-model instead of the original model. The first statistical parameter we chose for evaluating the meta-models performance is the correlation coefficient $R^2$ (eq. 6), classically used in statistic:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{\sum_{i=1}^{n} (\hat{y}_i - \bar{y}_i)^2}$$

(6)

Where $\bar{y}_i$ is the mean model response.

Finally, a particular attention was devoted to the analysis of the distribution of the residuals (the difference between model results and meta-model predictions). Especially, the residual plots can help avoiding inadequate meta-models and help adjusting the meta-model for better results. In general, it is assumed that the simplest meta-model that produces random residuals is a good candidate for being a relatively precise and unbiased model. In this study, residuals $\epsilon_i$ between model $y_i$ and meta-model $\hat{y}_i$ predictions are evaluated as follow:

$$\epsilon_i = y_i - \hat{y}_i$$

(7)

Two kind of analysis are possible, both performed in this study: (i) the analysis of the distribution of the residuals, which requires residuals to be normally distributed with mean zero (Jarque and Bera, 1980) and (ii) the analysis of the cumulative frequency distribution of the modulus of residuals (reported in the next section). This latter analysis provides an order of magnitude of the total uncertainty related to meta-model prediction, as discussed in the next sections.

### 2.3 Step 3: Uncertainty Quantification and Global Sensitivity Analysis

Considering the variety and the complexity of the geophysical mechanisms involved in tsunami generation, tsunami hazard assessment is generally associated with strong uncertainties (aleatory and epistemic). In PTHA, uncertainties are classically integrated in a rigorous way (Selva et al., 2016; Sørensen et al., 2012; Horspool et al., 2014) and quantified using the logic-tree approach (Horspool et al., 2014) and/or random simulations performed using the Monte-Carlo sampling of probability density functions of geological parameters (Horspool et al., 2014; Sørensen et al., 2012). An alternative and interesting approach is recently proposed by Selva et al. (2016), consisting in the use of an event tree approach and ensemble modelling (Marzocchi et al., 2015). Moreover, a new procedure was recently proposed by Molinari et al. (2016) for the quantification of uncertainties related to the construction of a tsunami data-base based on the quantification of elementary effects.
In this work we propose a classical methodology that could be adapted to analyse tsunamigenic regions with poor (or no) information on crustal characteristics and based on the classical uncertainty study steps (Saltelli et al., 2008; Saltelli et al., 2004; Faivre et al., 2013; Iooss and Lemaître, 2015). This methodology was already tested in other hydraulic context in recent years (Abily et al., 2016; Nguyen et al., 2015). At first, the variation intervals and the probability density functions (PDF) for the inputs affected by uncertainty must be defined for UQ. We remind that, for a random variable $X$ defined on an $[a,b]$ interval, a PDF $f_X(X)$ is defined as follows:

$$ P(X \in [a,b]) = \int_a^b f_X(X) \, dx $$

(8)

Most of the time, as in the present study, the probability distributions of random inputs is unknown. Hence, expert’s knowledge or the experimental evidence available is used as the basis for the PDF definition. The step 3.1 of our methodology requires propagating in the model the uncertainty associated to the input parameters with the objective of computing the PDF $f_y(Y)$ of the resulting quantities of interest $y$, as the maximum tsunami height employed in this study.

Finally, a better understanding of uncertainties can be achieved by analysing the contribution of the different sources of uncertainty to the uncertainty of the variables of interest (Step 3.2). Post-treatment procedures are used in order to rank the sources of uncertainty. It is important to note that this last step plays a crucial role. Indeed, the ranking results highlight the variables that truly determine the relevancy of the final results of the study. The methods employed for Step 3.1 and 3.2 are described in the next paragraphs.

### 2.3.1 Step 3.1: Uncertainty Propagation using Monte-Carlo method

The Monte-Carlo method used in this study requires random generation of input variables from their probability distributions. This step is the most complex and computationally demanding. In fact, the number of simulations increases with the increase of the uncertainty of each parameter. For this reason, alternative numerical techniques could also be used to reduce the computational time and yet preserve the quality of the statistical results (i.e. quasi Monte-Carlo screening methods). For instance, in this study, we used kriging meta-models (see section 2.2) instead of the original model to perform UQ using Monte-Carlo simulations.

The resulted sampling of a given size $N$ is a $N \times V$ matrix, $V$ being the number of uncertain parameters. Each row of the matrix $x^i = (x_{i,1},...,x_{i,V})$ represents a possible configuration of the input parameters of the model, that are, in this study, the fault parameters presented in Figure 2. Corresponding realizations of the output “$Y$” (the maximum tsunami water height in this study) are generated by successive deterministic simulations with each configuration of the inputs. Statistical estimators of the response $Y = (Y_1,...,Y_N) = (G(x^i))_{i=1,...,N}$ can therefore be computed from the output as follows:

$$ E[Y] = \mu_Y = \frac{1}{N} \sum_{i=1}^{N} G(x^i) $$

(9)
\[ Var(Y) = \frac{1}{N-1} \sum_{i=1}^{N} [G(x^i) - \mu_Y]^2 \]  

(10)

\[ \sigma_Y = \sqrt{Var(Y)} \]  

(11)

Where \( E[Y] = \mu_Y \), \( Var(Y) \) and \( \sigma_Y \) are, respectively, the mean, the variance and the standard deviation of the response \( Y \) given by the model \( G \). These statistical moments are useful for both the uncertainty propagation and the uncertainty sensitivity analysis (see next paragraph). It must be noted that \( \sigma_Y \) is classically employed for the quantification of the global uncertainty issued from Monte-Carlo simulations.

The convergence order of the Monte-Carlo sampling method is given by the Central Limit Theorem (Faivre et al., 2013) as \( O\left( \frac{1}{\sqrt{N}} \right) \), implying that a large amount of simulations are necessary to obtain convergent statistics.

2.3.2 Step 3.2: Global sensitivity analysis

The role of the sensitivity analysis is to determine the strength of the relation between a given uncertain input parameters and the model outputs (Saltelli et al., 2004). In this study, we focused our attention on Global Sensitivity Analysis approaches which rely on sampling based methods for uncertainty propagation, willing to fully map the space of possible model predictions from the various model uncertain input parameters and then, allowing to rank the significance of the input parameter uncertainty contribution to the model output variability (Baroni and Tarantola, 2014). The objectives with this approach are mostly to identify the parameter or set of parameters which significantly impact models outputs (Volkova et al., 2008; Iooss et al., 2008).

In this study, the GSA approach we focused on is the Sobol index computation, that considers the output hyperspace \( x \) as a function \( Y(x) \) and performs a functional decomposition (Iooss and Lemaître, 2015; Iooss, 2011) or a Fourier decomposition (FAST method) of the variance (Saltelli, 2002; Saltelli et al., 1999). With respect to deterministic methods (i.e. Morris, 1991), probabilistic GSA approaches relying on Sobol index computation go one step further, by quantifying the contribution to the output variance of the main effect of each input parameter (Sobol, 2001; Saltelli et al., 1999; Saint-Geours, 2012; Sobol, 1993). The definition of Sobol Indices is a result of the ANOVA (ANalysis Of VAriance) variance decomposition. To be able to write the variance decomposition, some mathematical hypothesis are required (see i.e. Saltelli et al., 2008, or Faivre et al., 2013), briefly recalled here.

Let us consider an integrable model \( G \) on the domain \( A = A_1 \times \ldots \times A_v \). It exists an unique decomposition of the model:

\[ G(Y) = f_0 + f_1(Y_1) + ... + f_m(Y_m) + f_{1,2}(Y_1, X_2) + ... + f_{v-1}(Y_{v-1}, Y_v) + ... + f_{1,...,v}(Y_1,..., Y_v) \]  

(12)
Where all the functions $f_i$ are mutually orthogonal, implying that $\left\langle f_i \left| f_j \right. \right\rangle = 0$ if $i \neq j$, with the scalar product defined as follows:

$$\left\langle f \left| g \right. \right\rangle = \int_A f(x)g(x)\pi(x)dX$$ (13)

Where $\pi$ is a probability distribution defined on $A$. Under these assumptions, given a set of $V$ independent uncertain parameters $X$, the variance of a response $Y = G(X)$ can be calculated, using the total variance theorem, as follows:

$$\text{Var}(Y) = \sum_{i=1}^{V} D_i(Y) + \sum_{i<j}^{V} D_{ij}(Y) + \ldots + D_{12,V}(Y)$$ (14)

Where $D_i(Y) = \text{Var}[E(Y|X_i)]$ and $D_{ij}(Y) = \text{Var}[E(Y|X_i, X_j)] - D_i(Y) - D_j(Y)$ and so on for higher order interactions. $E(Y|X_i)$ is the $Y$ conditional expectation with the condition that $X_i$ remains constant. The so-called “Sobol’ indices” are obtained as follows:

$$S_i = \frac{D_i(Y)}{\text{Var}(Y)}, \quad S_{ij} = \frac{D_{ij}(Y)}{\text{Var}(Y)}, \quad \ldots$$ (15)

These indices express the share of variance of $Y$ (the maximum tsunami height) that is due to a given combination of input parameters (the seismic sources reported in Figure 2). For instance, $S_i$ is the first order Sobol index. The number of indices grows in an exponential way with the number $V$ of dimensions. So, for computational time and interpretation reasons, (Homma and Saltelli, 1996) introduced the so-called “total indices” or “total effects” that write as follows:

$$S_{Ti} = S_i + \sum_{i<j} S_{ij} + \sum_{j<i,k>1} S_{ijk} + \ldots = \sum_{i \in S} S_i$$ (16)

Where $\neq i$ are all the subsets of $\{1,\ldots,V\}$ including $i$.

GSA approaches are robust, have a wide range of applicability, and provide accurate sensitivity information for most models (Adetula and Bokov, 2012). Moreover, even if they are theoretically defined for linear mathematical systems, it was demonstrated that they are well suited to be applied with models having nonlinear behaviour and when interaction among parameters occur (Saint-Geours, 2012), as in the present study. For these reasons, these indices were already adopted for the analysis of bi-dimensional hydrodynamic simulations in urban areas (Abily et al., 2016) or of complex coastal models including interactions between waves, current and vegetation (Kalra et al., 2018) and they seem well suited for the present work. For the computation of Sobol’ indices, a large variety of methodology are available, as the so-called "extended-FAST" method (Saltelli et al., 1999), already used in previous studies by IRSN (Nguyen et al., 2015). In this study, we used the methodology proposed by Jansen et al. (1994) already implemented in the open source sensitivity-package R (Pujol et al.,
3 Example of application: building a numerical data-base of tsunamis generated by the Azores Gibraltar Plate Boundary (AGPB) and impacting the French Atlantic Coast

3.1 Design data-base for meta-models construction

In this methodological work, we considered a widened AGPB tsunamigenic area as a test case for DTHA. We chose to explore as largely as possible the potential tsunami height along the French Atlantic Coast generated by earthquakes from 34° to 40°N and from 18° to 7°W, encompassing to the East the southern part of Portugal down to Morocco, and reaching the oceanic sea-floor west of the Madeira toe rise (as reported in Figure 3). Because the design database is a learning base for meta-modelling, the range of variation of the inputs parameters (column “Design database” in Table 2) need to be large in order to cover both a wide range of earthquake scenarios as well as most of the supposed possible sources of the 1755 Lisbon event and earthquakes coming from the following catalogues: http://www.isc.ac.uk; http://earthquake.usgs.gov; http://www.globalcmt.org; http://www.bo.ingv.it/RCMT/. Thus, if correctly estimated, meta-models will be able to reproduce the model behaviour for a large range of variations of the seismic inputs parameters, including physical scenarios from geological studies of the zone.

In order to build the design database, fault parameters as defined in section 2.1 and Table 2 were sampled randomly and independently with the Monte-Carlo method and supposing uniform distributions. The uniform distribution was chosen in order to build meta-models able to reproduce tsunamis heights generated by various tsunamigenic sources with the same accuracy. The resulting earthquake magnitudes are computed using the sampled parameters with equation 2. The shear modulus chosen for the magnitude estimation is a constant value assumed to be equal to 30 GPA. This design database is a matrix which associates to a given combination of fault parameters estimates the maximum simulated water height at each point of the numerical grid and also at four selected locations along the French Atlantic Coast called gauges (Table 3 and Figure 3), namely, from North to South, “Saint-Malo”, “Brest”, “La Rochelle” and “Gastes”. The maximum tsunami water height is the relevant parameter when estimating tsunami hazard. The design database contains 5839 scenarios split in a training set of 4672 scenarios and a testing set of 1167 tsunamis scenarios, used for the meta-model validation and the residual analysis. The water height characteristics associated to these scenarios are reported in Table 4.

Finally, the meta-models are built using the open-source R-packages DiceKriging (Roustant et al., 2012), according to the equations 21, 22 and 23 reported in the APPENDIX 1. Each meta-model is a function able to compute the maximum tsunami water height at the gauge location for a given set of seismic source parameters (strike, length, dip, rake, width, slip, longitude, latitude and depth). Obviously, the input parameters should be included in the parameter range used for the meta-model construction and reported in Table 2.
3.2 Meta-models validation

3.2.1 Meta-models uncertainty

As reported in the section 2.2, the training set is split into $K$ folds of approximately equal size and a model is fitted from the data and validated on the fold $A_k$. This procedure was automatically managed by R-package sensitivity for the four Atlantic Gauges and the results in terms of a criterion of quality $L$ are reported in Table 5. The mean $MSE$ computed is very low (varying from a minimum value of 0.004 for Saint Malo to a maximum value of 0.029 for La Rochelle), indicating that the kriging meta-model is a good emulator choice for reproducing the CEA-tsunami code behaviour. Finally, for each gauge, we constructed a kriging meta-model using all the scenarios of the training set data base.

Further, the residual analysis was performed by comparing the model results with the meta-models predictions for the validation set parameters. The objectives of this analysis are (i) to test the ability of the selected meta-model to reproduce model results for another set of parameters (outside of the training data-set space), by the computation of the correlation coefficient and by the analysis of the residuals distribution (eq. 7), and (ii) to estimate the uncertainty associated to the use of a meta-model instead of the original model. If the validation results are not satisfactory, this would suggest that other simulations need to be performed to complete the design data-base and reduce the excessive bias derived from a rough sampling of the input parameters.

The computation of the correlation coefficient shows a good agreement between the original model results and meta-models, for the four selected gauges (see Table 5 and Figure 5). $R^2$-values are generally high (the minor values for “La Rochelle”, of 76%, is satisfying considering the methodological objective of the study and the analysis reported in Marrel et al. (2009) and Storlie et al. (2009), even if we can observe that the strongest water height from the original model are slightly underestimated by the meta-models for the validation set data base (Figure 5). Considering the strong differences (Table 4) between the “maximum” water height and the “$\mu_m + 2\sigma_m$” water height of the design database ($\mu_m$ and $\sigma_m$ being, respectively, the mean and the standard deviation of the tsunami water height), which indicate that the lower water height are largely represented, it is not surprising that the meta-models predictions are more accurate for lower tsunami height than for extreme water height. These results are confirmed by the computation of the distribution of the absolute residuals (the modulus of Eq. 7) and of the density frequency distribution of the relative residuals (Eq. 7), as reported in Table 5 and Figure 4. This analysis shows that the residuals are normally distributed with mean zero (“$\mu_r$”) and lower standard deviation (“$\sigma_r$”), which is satisfying (Jarque and Bera, 1980), and that the maximum residuals are high for the fourth gauge (Table 5), probably for the reasons reported above.

In conclusion, the residuals analysis confirms that, globally, the meta-models constructed for the four French Atlantic gauges are able to reproduce the model behaviour (the residuals are globally low with the “$2\sigma_r$” of few centimetres), and they can be used for the construction of the numerical tsunami database. Nevertheless, we consider that when meta-models are used for the predictions of the tsunamis height instead of the original model, an uncertainty must be considered and applied to the meta-modelled water heights. According to results reported in Table 5 and Figure 5, an
uncertainty equal to “$+/−2\sigma_r$” calculated with the validation set should be conservative, considering the global residual trend (Table 5, Figure 5).

3.2.2 Comparison with Allgeyer et al. (2013) results along the French Atlantic Coast

Another possible validation of the meta-models consistency is the comparison with a previous study from Allgeyer et al. (2013). This comparison is of interest in order to confirm the ability of the constructed meta-models to reproduce the order of magnitude of the modelled tsunami height at a given location. In their study, the authors analysed the impact of a Lisbon-like tsunami on the French Atlantic Coast through numerical modelling. Especially, the authors focused on the simulated maximum tsunami water height in the North Atlantic associated to three different sources for the 1755 events derived from Johnston (1996), Baptista et al. (2003) and Gutscher et al. (2006), for a total of five tsunami scenarios. The same scenarios were simulated with our meta-models for the four French Atlantic Gauges. Even if the results obtained with meta-models (see Table 6) cannot be really compared with results from the authors (Allgeyer et al., 2013), which are obtained with a more refined grid, spacing from 1’ to 0.3” near to the “La Rochelle” harbour, the general pattern reported in Allgeyer et al. (2013) is confirmed. Especially, the authors pointed out that the coastal area offshore French Brittany (from 46 °N to 48 °N) were impacted by amplitudes of 0.6 to 0.8 m for the Johnston source (1996) and not exceeding the 0.5 m for the Baptista et al. (2003) and the Gutscher et al. (2006) sources. These results are consistent with those obtained with our meta-models for the Brest and La Rochelle gauges which represent this coastal area (Table 6). As a consequence, the results obtained with meta-models by using some physical sources by literature seem reasonable, physically speaking.

3.3 Construction of a tsunami data-base for the French Atlantic Gauges

Starting from the validated meta-models we finally build a database of tsunamis generated by the considered AGPB area at four French Atlantic Gauges. This numerical data-base should permit to cover all the possible tsunamis height impacting a given area and then to assess the tsunami hazard level (MCS_h). Moreover, in a context with strong uncertainties on the seismic source parameters, this methodology should also permit to verify if the tsunamis height associated with the MCS_p parameters are robust with respect to the area to protect.

AGPB is a zone prone to tsunamis and its investigation already lead to the publication of tsunami catalogues for the zone (Baptista and Miranda, 2009; Kaabouben et al., 2009). However, for the purpose of this methodological paper, the AGPB area modelled here, which encompass different seismotectonic domains, was split in a very simplistic way into two main seismic source zones. This was done to represent a western domain where normal to transtensive earthquakes mainly occur within an oceanic crust and an eastern domain to the East where reverse to transpressive earthquakes mainly occur on an oceanic crust and a thicker continental crust (Buform et al., 1988; Molinari and Morelli, 2011; Cunha et al., 2012). As a first order analysis, we considered the Western domain west of the 10°W meridian and the Eastern domain east of this meridian, coinciding roughly with the base of the continental slope facing the Portuguese coastline. Even if the zonation adopted in our study is not in agreement with the actual knowledge of the deep structure of the Gulf of Cadiz and the Gloria Fault zone as
given by Martínez-Lortient et al. (2014) and Batista et al. (2017), such a simplified zonation represents a first order approach of the AGPB seismotectonics, consistent with the global objective of the study aimed at illustrating a methodological approach to quantifying all possible tsunamis heights that can potentially impact a given area. A more accurate consideration of faults parameters that are better constrained in some regions of the AGPB area would be more in line with the work presented by Roshan et al. (2016) or with a complete and robust deterministic MCS_p approach.

The main fault characteristics considered to perform earthquake scenarios in both eastern and western domains are reported in Table 2, representing our interpretation of data contained in (Buform et al., 1988; Molinari and Morelli, 2011; Cunha et al., 2012). The considered seismogenic thickness takes into account the depth of the observed seismicity and crustal structure for each zone as well as a part of the upper mantle that can potentially be mobilised during major earthquakes: the Atlantic shelf seismogenic thickness is considered to be of the order of 20 km (after Baptista et al., 2017), and 60 km for the European shelf (after Silva et al., 2017). The fault parameters are considered uniformly distributed and are randomly sampled in their range of variation. We finally filtered the resulting database according to an aspect-ratio criterion, allowing the ratio between length and the width of the faults not to exceed the value of 10, which correspond to an upper bound of what is observed in nature (Mc Calpin, 2009). The final database contains nearly 50,000 tsunami scenarios, resulting in earthquake magnitudes varying from 6.7 to 9.3 (Table 2), depending on the explored earthquake sources characteristics and calculated from Eq 1. This range of magnitudes is consistent with the magnitude range of the design database. It must be noted that the higher magnitudes considered for the AGPB (higher than the maximum value estimated for the 1st November 1755 earthquake of 8.8, according to Johnston (1996), correspond to some extreme scenarios which represent less than 0.2% of the total scenarios of the numerical data-base. However, our choice is to not ignore these very unlikely scenarios. In fact, the objective of this work is to propose a methodology which could be applied in a context of poor knowledge and could permit to ensure that MCS_p is “really” the appropriate scenario to consider for a given location. MCS_h is deduced from the tsunamis height distribution at a given location. Depending on the specific hazard target (civil or industrial facilities), and its location respect to the source zone, the end-user of this methodology will need to decide which level of water height to choose from the obtained distribution (MCS_h).

The resulting global database (Table 2) is composed by the union of the western and the eastern databases scenarios. The tsunamis height characteristics associated to the four gauges are reported in Table 7. It must be noted that the maximum tsunami height of the global database are lower than the tsunamis height of the design database and that the convergence of statistics (mean water height) is largely achieved, as reported in Figure 6. The achievement of the convergence of numerical results is an important aspect of the adopted methodology and suggests that it is not necessary to further explore the space of inputs parameters in order to assess CMS_h: statistically, the distribution of tsunami heights is representative of the source variability in the defined space of parameters.

In conclusion, the built tsunamis database (the global database) should represent a wide sample of tsunami heights potentially generated by the AGPB (the Western and the Eastern domains). The database is analysed in the next section with a focus on DTHA. At first, the resulting tsunami height distribution is analysed in order to propose a hazard level for the
French Atlantic Gauges (CMS_h). Then, this choice is discussed in the light of tsunami heights higher than the proposed hazard level (section 4.2) and of tsunamis heights generated by a MCS_p-like scenarios (section 4.3).

4 Analysis of the numerical tsunamis database

4.1 Assessment of a hazard level for the French Atlantic Coast (MCS_h)

The distribution of tsunami heights resulting from each considered tsunamigenic source is reported and compared in Figure 7 for each gauge. It can be observed that the tsunami heights generated by the western shelf are globally lower than those generated by other sources. This is not surprising given the different depths explored for the two regions (see Table2). Indeed, the highest tsunami heights resulting from the global database are globally generated by the eastern shelf for both the southern and the northern gauges.

From an engineering point of view, the tsunami hazard “MCS_h” can be directly deducted from Figure 7 (black line). Roshan et al. (2016) proposed to use a hazard reference level based on the mean tsunami height resulting from their distribution. However, as reported in section 3.3, the constructed tsunami data-base integrates all the “likely” and “unlikely” tsunami scenarios from AGPB. As a consequence, we prefer to set the tsunami hazard reference level at the value “$\mu_m + 2\sigma_m$” of the modelled water height for each gauge and to consider an uncertainty associated to this level equal to the +/-2$\sigma_r$ residual (summarized in Table 8 for each gauge). This choice needs to be justified on a case by case basis. In this specific example the choice is guided, firstly, by considering the shape of the tsunami heights distribution resulting from UQ (Figure 7, black line). In fact, the distributions for each gauge are centred on the lower tsunami heights while the maximum modelled tsunami heights are very large with respect to the tsunami statistical parameters of the distribution (reported in Table 8). As a consequence, this choice appears numerically reasonable in a context of DTHA as it considers both the low tsunami height and the maximum modelled tsunami height. Secondly, considering the rough approximation of the earthquake input parameters (and the consequently wide range of seismic magnitude distributions) used for the construction of the global database, this hazard level appears more conservative in a context of DTHA.

The proposed hazard level corresponds to nearly 200 tsunami scenarios (+/- 0.01 m respect to the chosen value) of the data-base, located all over the AGPB but preferentially in the eastern part for the “Saint-Malo” and the “Brest” gauges and exclusively in the eastern part for the southern gauges for “La Rochelle” and “Gastes” gauges (Figure 8). The MCS_h scenarios are associated with earthquake magnitudes in the 7.5 to 9 range for all the gauges, with a mean of 8.47 for the northern gauges and of 8.67 for the southern gauges. The robustness of this choice is analysed in the next sections. Especially, it is of interest to analyse the strongest tsunamis of the numerical database (see section 4.2) and to evaluate if this hazard level is robust with respect to results which could be obtained with a more classical scenario-based approach “MCS_p” (see section 4.3).
4.2 Focus on the strongest tsunamis of the numerical database

As reported in Table 8, there are strong differences, for all the gauges of the numerical database, between the maximum simulated tsunami height and the \( \mu_m + 2\sigma_m \) water height and the hazard level chosen in this study \( \text{MCS}_h \). As a consequence, from a methodological point of view, and with the objective of assessing the tsunami hazard level from the UQ study, it is of interest to understand which scenarios are stronger than the proposed hazard level.

With this aim, we first analysed the magnitude distribution \( M_w \) of the tsunami scenarios of the global database (grey curve in Figure 9a) generating tsunamis heights higher than the proposed hazard level at each gauge (coloured lines in Figure 9a). Not surprisingly, the strongest tsunami heights correspond to the strongest seismic events of the database (Figure 9a). In particular, more than 80\% of the tsunami heights higher than the chosen hazard level \( \mu_m + 2\sigma_m \) are associated with a seismic magnitude higher than 8.5. These tsunami events completely represent the tail of the earthquake magnitude distribution (the grey histogram in the Figure 9a). Nevertheless, some discrepancies are observed between the results of the southern gauges (La Rochelle and Gastes) and the results obtained for the northern gauges (Brest and Saint-Malo). In fact, even if it seems that only the very strong earthquakes (magnitude higher than 8.5) can generate a tsunami height higher than \( \mu_m + 2\sigma_m \) for the middle-south of Atlantic French coast (more than the 95\% of the tsunami scenarios of the sample), this is more ambiguous for the northern Gauges. In this case, nearly 30\% of the tsunamis higher than the hazard level can be generated by an earthquake of magnitude in the range [7.8 – 8.5], which includes the upper range of the western shelf.

As shown in Figure 9b, 50\% of the simulated tsunamis in this [7.8 – 8.5] magnitude range generating tsunami heights greater than chosen hazard level \( \mu_m + 2\sigma_m \) originate in the western shelf. In fact, the northern gauges (Saint-Malo and Brest) are practically the only ones exposed to tsunamis generated by the western shelf.

These results are interesting and show that the strongest water height obtained through UQ (stronger than \( \text{MCS}_h \)) are not all associated with the strongest magnitudes, which are typically considered in the \( \text{MCS}_p \) approach. On the contrary, most of the strongest tsunamis heights at the northern gauges could be generated by the western shelf, for seismic magnitudes in the range [7.8 – 8.5]. In conclusion, these results also suggest that, beyond earthquake magnitudes, the position and the orientation of the faults are influent parameters. In terms of hazard level, which is the original objective of this analysis, one can consider, at first, that tsunamis higher than the chosen hazard level are mainly generated by seismic magnitudes stronger than 8.5, even if some rare combinations of tsunami input parameters can lead to higher tsunami heights.

4.3 Comparison with a classical \( \text{MCS}_p \) approach associated to a Lisbon-like 1755 tsunami scenario

As reported in the introduction, DTHA is classically assessed by means of considering particular source scenarios (usually maximum credible scenario) and the associated maximum tsunami height is generally retained as hazard level \( \text{MCS}_p \). A more sophisticated method was recently proposed by Roshan et al. (2016). The authors tested around 300 tsunami scenarios in a [8 – 9.5] magnitude range associated with various faults potentially impacting the Indian coast.
Finally, the authors suggested that an appropriate water level for hazard assessment (e.g. mean value or mean plus sigma value) should be retained. They proposed the mean value of the simulated water heights, as test, by considering that this value may need to be revisited in the future.

In the case of the tsunamis hazard associated with the AGPB, the 1755 Lisbon tsunami is the classical reference scenario. For the critical analysis of the global numerical database we decided to complement our study with two specific and nearly deterministic scenarios considered as the most-likely sources generating the Lisbon 1755 tsunami, namely the Gorringe and Horseshoe structures (Buñorn et al., 1988;Stich et al., 2007;Cunha et al., 2012;Duarte et al., 2013;Grevemeyer et al., 2017). Both structures were modelled taking into account available maps (Cunha et al., 2012;Duarte et al., 2013) and fault parameters (Stich et al., 2007;Grevemeyer et al., 2017) summarized in Table 2. These scenarios have been chosen because they are representatives of the MCS_p approach for the study zone. They permit to evaluate the global tsunami database, which was designed by roughly zoning European (eastern) and Atlantic (western) shelves. In fact, the tsunami hazard induced by the Horseshoe scenario, located at the limit between these two zones of the global database, may be partly contained in the European contribution of the database. On the contrary, the Gorringe scenario represents a pure “European like” scenario within the Atlantic zone, hence excluded by the global database.

For the computation of the tsunami heights associated with these scenarios, fault parameters are considered uniformly distributed and are randomly sampled in their range of variation (Table 2), for a total of nearly 10,000 tsunami scenarios. The magnitude range associated with these tsunami scenarios varies from 7.7 to 8.9 and the hazard level associated to these faults corresponds to the “μ_m” modelled water height. This hazard level for MCS_p approach was arbitrarily chosen, considering that the philosophy of this kind of approach is to take into account only the worst possible scenarios sources. In this sense, this value is in agreement with the choice reported in Roshan et al. (2016).

In Table 8 we summarized the tsunami hazard height from the global database and from Gorringe and Horseshoe faults for each gauge, with the associated uncertainty (corresponding to the residuals +/- 2σᵣ). These results suggest that the strongest tsunamis are globally generated by the Gorringe bank and that the proposed hazard level from the total database (MCS_h) is representative of the scenarios based tsunamis (MCS_p), once the uncertainty associated to meta-model predictions is considered (see Figure 10). In conclusion, the proposed hazard level resulting from UQ seems reasonable in light of the results obtained through a scenario-based-like approach, at least for the tsunamis impact along the French Atlantic coast due to sources off of Portugal.

### 4.4 Results from sensitivity analysis: the influence of the seismic-source parameters: application of Step 3.2

Homma and Saltelli (1996) introduced the total sensitivity index which measures the influence of a variable jointly with all its interactions. If the total sensitivity index of a variable is zero, this variable can be removed because neither the variable nor its interactions at any order have an influence on the results. This statistical index (called Sobol index “ST” in this paper, Eq. 16), is here of particular interest in order to highlight the earthquake source parameters that mostly control the tsunamis height at each tested gauge. In Figure 11, we reported the total Sobol index for the four meta-models of the French...
Atlantic Gauges computed with the methodology proposed by Jansen et al. (1994) using the sensitivity-package R (Pujol et al., 2016). The accuracy of Sobol indices performed with Jansen’s method depends on the number of model evaluations. For instance, in this study, we performed nearly 20 000 simulations using meta-models for the computation of Sobol indices. Results show that the slip parameter is globally the most-influencing parameter for all the French Atlantic Gauges meta-models, which is quite obvious considering that the fault-slip directly conditions the ocean floor deformation and hence the tsunami amplitude. However, Figure 11 suggests that the most influencing parameters for the four gauges are slightly different, depending on their location. One can differentiate results obtained for the southern gauges (i.e. La Rochelle & Gastes) from those obtained at northern gauges (i.e. Saint Malo & Brest):

- For the southern gauges, width is the second most relevant parameter, especially at La Rochelle where it is almost as important as the slip parameter. Other important parameters are length and rake, suggesting that for these gauges, fault source parameters in terms of magnitudes (depending on width, slip and length according to equations 1 and 2) and kinematics are the most important in generating hazard;

- For the northern gauges, apart from slip, strike, width and rake are also important, but slightly less than the longitude parameter. This means that the location of the source is here of major importance. A possible physical reason could be associated to the lack of the natural barrier composed by the north of Spain and Portugal which protects southern gauges from AGPB related tsunamis compared to northern gauges. As a consequence, the northern gauges should be more exposed to hazard in comparison to southern gauges, located in the shadow of Portugal and Spain.

In conclusion, these results also confirm that, beyond earthquake magnitudes, the position and the orientation of the faults are influents parameters, in agreement with the analysis of the previous paragraph (see 4.2). Thus, for a given magnitude, a tsunami generated by a “well located-oriented” fault would be potentially more hazardous, at least for the sites along the northern French Atlantic Coast. In other words, the French Atlantic coast is in the shadow of the tsunamis generated by the stronger continental shelf sources and more sensitive to the tsunamis generated in the oceanic shelf.

5 Conclusions and perspectives

The methodological research work presented in this paper was performed in order to test the interest of UQ for the assessment of the DTHA generated by earthquakes. We propose a new methodology for DTHA, consisting in the assessment of tsunami hazard through the analysis of all the possible tsunami heights at a given location (MCS_h). This concept goes beyond the definition of the Maximum Credible Scenario source parameters (MCS_p) classically reported in the literature (JSCE, 2002; Lynett et al., 2016) and employed for DTHA. MCS_h is evaluated through UQ allows the exploration of the tsunamigenic potential of a seismic zone in a rigorous and complete way. It can be considered as an extension of the previous innovative work from Roshan et al. (2016), which first improved the classical deterministic approach by focusing its analysis on some selected earthquake source parameters.
The methodology is based on the use of kriging meta-models that intensively explore all the possible tsunamis generated by a given seismic area. It is the first time, to our knowledge, that meta-models are used in this way in the context of DTHA (but also for PTHA). The meta-models are first constructed through a uniform exploration of the space of the inputs parameters of the seismic area and then tested and validated through a series of statistical indicators (correlation coefficient, residuals analysis, MSE). These performance-tests allowed us to conclude that, even if the maximum residuals (differences between model and meta-model results) are very high for the four selected locations along the French Atlantic Coast (see Table 5), the meta-models are able to reproduce the model behaviour and they can be used for the construction of the numerical tsunami database. Nevertheless, when meta-models are used instead of the original model for the predictions of the tsunami heights, we propose to add an uncertainty corresponding to the “$\pm 2\sigma_r$” residual value measured at each gauge. Moreover, the realism of the tsunami height simulated with the meta-models was also ensured through a comparison with the numerical results presented in Allgeyer et al. (2013) for the French Atlantic Coast. Thus, the constructed meta-models could be employed in a further study to roughly evaluate the impact of other seismic scenarios from the AGPB and impacting the French Atlantic Coast.

In the example of application presented in this work we built our meta-models using a design data-base of nearly 5 000 tsunami scenarios. The number of simulations is large and required intensive parallel computing using GPU. However, considering the large number of model evaluations necessary for UQ (more than 50 000) and GSA (more than 20 000), the meta-models appear as the more adapted tool for the proposed methodology.

We test this methodology by building a numerical database (of nearly 50,000 tsunami scenarios) of tsunamis potentially generated by the AGPB and impacting the French Atlantic Coast at four sites of interest. As frequently reported in the paper, this is only a test-case and all the results and the analysis presented in section 4 are an example of how an end-user of the methodology should use these results for a DTHA application. This database should permit to explore the numerous uncertainties related to the AGPB characteristics. Considering the convergence of the mean modelled tsunami height, it is not necessary to further explore the AGPB inputs parameters in order to refine the assessment of the CMS_h for the French Atlantic Gauges since statistically we have shown that the resulting tsunami height distribution is representative of the source variability. This may not be the case, however, for sites located closer to the AGPB region. In such regions a more refined zonation is necessary. However, the proposed methodology remains unchanged.

Tsunami hazard (MCS_h) was assessed directly from the tsunami height distribution of the database (see section 4.1). Considering the rough zonation of the AGPB and the shape of the tsunami heights distribution we propose in this case to attribute to each French Gauge a tsunami hazard level equal to the modelled tsunami height “$\mu_m+2\sigma_m$” with an associated uncertainty equal to the residual “$\pm 2\sigma_r$”. To validate this choice, we analyse the strongest tsunamis (higher than the proposed hazard level) of the distribution. In our example, this analysis permits to conclude that, globally, the position and the orientation of the fault are influent parameters. In other words, for a given magnitude, a tsunami generated by a “well located-oriented” fault would be potentially more hazardous, at least for the sites located along the northern French Atlantic Coast. In this sense, the UQ study permitted to explore some rare combinations of seismic inputs parameters generating
higher tsunamis level (see Figure 9b). Moreover, the UQ permits to underline that globally the tsunamis generated by the western shelf are largely less impacting than the tsunamis generated by the eastern shelf, on average. Furthermore, GSA performed with Jansen’s method permits to confirm these first results. Especially, the magnitude generating the tsunamis in the AGPB (depending on width, slip and length) is more relevant depending on the location of the gauge (the impacted zone). For instance, only the earthquakes inducing large displacements of the sea-floor (large slip) are potentially capable to affect the middle/south of France. However, concerning the gauges located to the north of the French Atlantic Coast (Brest and Saint-Malo), other influencing parameters are, apart from the slip, the location (longitude) and the orientation (strike and rake parameters) of the fault sources. These results are also in agreement with results presented in (Allgeyer et al., 2013).

The data base was also tested through a comparison with the tsunamis generated by a Lisbon-like 1755 earthquake (paragraph 4.3). The latter representing a comparison between the global results from UQ (MCS_h) and a more classical MCS_p approach. Globally, the hazard level issued by the total database seems robust and representative of the tsunamis from Gorringe and Horseshoe banks once the uncertainty of numerical results is considered (see Figure 10), at least for French gauges off the Atlantic coast. Even if this work is mainly methodological, this results seems to confirm that the proposed MCS_h is adapted to the French Atlantic Coast as the tsunamis height from these scenarios are the strongest of the tsunamis heights resulting from the global data-base. However, to be conclusive, the same analysis should be conducted with other MCS_p hypothesis for the AGPB. Moreover, it must also be noted that the choice of the hazard level is arbitrary and it depends on the chosen area, the available information about potential tsunami sources and the analysis of the obtained tsunami heights distribution.

Tsunami hazard is analysed only off the French Atlantic Coast and by using, as input data for meta-model’s construction, the deterministic simulations performed with a rough computation grid-resolution of 2” degrees (~3.6 km) which are not adapted to reproduce the influence of bathymetry on tsunami propagation in the vicinity of the coastal areas. For an operational DTHA along the French Atlantic Coast, it should be necessary, at first, to improve the actual numerical model in order to better represent the tsunamis run-up and the inundated areas, with a more accurate bathymetric grid. Then, the meta-models could be constructed with the methodology reported in section 2 and with a design data-base permitting to explore as largely as possible the tsunamigenic potential from AGPB. Once validated, these meta-models could be used for DTHA as reported in section 3.3. However, in order to really assess the hazard level for the French Atlantic Coast, the constructed global data-base should be compared, at least, to the main MCS_p hypothesis for this area, to ensure that the chosen hazard level is not underestimated with respects to these scenarios.

In perspective, for an operational application of the proposed methodology to locations closer to the source zones (i.e. Portugal or Spain, Morocco), the sensitivity analysis will probably show that slip, length and width are the dominant parameters. In such cases, a more in-depth source analysis is necessary and the research performed in the region should be carefully considered (i.e. (Vilanova and Fonseca, 2007; Woessner et al., 2015; Matias et al., 2013; Omira et al., 2016; Omira et al., 2015)). However, uncertainties will always remain. Thus even in such cases, where more refined databases need to be established, the proposed approach should be of interest. In fact, as shown by the Fukushima accident, it is today necessary
to go beyond the classical MCS_p approach and to focus on all the possible tsunami heights at a given location (MCS_h). In this sense, we consider that the modelled tsunami heights of our data-base are probably already representatives of the tsunamis heights generated by these scenarios for the French Atlantic Coast.

Finally, other analysis would be of interest for the improvement and the application of the methodology. At first, it should be possible to build more meta-models, in order to cover other area in locations where historical data from the 1755 Lisbone-tsunami are available (i.e. Santos and Koshimura, 2015; Baptista et al., 1998), thus permitting to further validate the constructed meta-models. Then, it would be of interest to test other hypothesis during the UQ step necessary for the assessment of the hazard (MCS_h), as different probability distribution of inputs parameters, a more realistic non-uniform slip distribution, considering its impact on tsunami heights (Geist and Dmowska, 1999), or, to analyze other approximation of the fault geometry.

As perspective, the example presented in this study for the AGPB suggests that it could be of interest to introduce the meta-models in the systems developed for tsunami early warning, considering the low computational time inherent to this statistical tool.
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APPENDIX 1 – UNIVERSAL KRIGING EQUATIONS

Originally coming from geosciences (Krige, 1951) and having become the starting point of geostatistics (Matheron, 1963), Kriging is basically a spatial interpolation method. However, starting from previous work of Sacks et al. (Sacks et al., 1989), many works dedicated to Kriging as a surrogate to computer experiments have been published (Welch et al., 1992; Koehler and Owen, 1996; O’Hagan, 2006). As a consequence, useful tools have been developed to build-up kriging meta-models in a single point for multi-variables complex problems. The theoretical background for kriging-metamodel design used in this study is fully detailed in (Roustant et al., 2012), which describes the R-packages DiceKriging and DiceOptim that we used for meta-models construction. We limited here to report the basic hypothesis of the methodology for the interested reader, directly issued from (Roustant et al., 2012).

Let us assume that the model response \( y \) is assumed to be a deterministic real-valued function of the \( d \)-dimensional variable \( x = (x_1, ..., x_d) \in D \subset \mathbb{R}^d \). \( y \) is assumed to be one realization of a square-integrable random field \( \left(Y(x)\right)_{x \in D} \) with first and second moments known up to some parameters. Let us consider that \( X = \{x^{(1)}, ..., x^{(n)}\} \) denotes the points where \( y \) has already been evaluated, and denote \( y = y(x^{(1)}, ..., x^{(n)})^T \) the corresponding outputs. For any \( x \in D \), the aim of Kriging will be to optimally predict \( Y_x \) by a linear combination of the observations \( y \). Especially, kriging-modelling treats the deterministic response \( y(x) \) as a realization of a random function \( Y(x) \), including a regression part and a centered stochastic process:

\[
Y(x) = f(x) + Z(x)
\]  

(17)

Where the deterministic function \( f(x) \) provides the mean approximation of the computer code.

For Universal Kriging (UK) employed in this study, \( Y(x) \) is assumed to be the sum of a known deterministic trend function \( \mu : x \in D \rightarrow \mu(x) \in \mathbb{R} \) and of a centered square-integrable process \( Z \):

\[
Y(x) = \mu(x) + Z(x)
\]  

(18)

\[
\mu(x) = \sum_{j=1}^{p} \beta_j f_j(x) \quad (p \in \mathbb{N} - \{0\})
\]  

(19)

Where \( f_j \) are fixed basis functions, \( \beta_j \) are unknown real coefficients and \( x \) is the \( d \)-dimensional variable. UK consists in deriving best linear predictions of \( Y \) based on the observations \( Y(X) \) while estimating the vector \( \beta = (\beta_1, ..., \beta_p)^T \). It must be noted that the stochastic part \( Z(x) \) is a Gaussian centered process fully characterized by its covariance function. \( Z \)'s covariance kernel \( C : (u,v) \in D^2 \rightarrow C(u,v) \in \mathbb{R} \) is known, considering that \( X = \{x^{(1)}, ..., x^{(n)}\} \) is known. By assuming \( Z(x) \) second order stationary, the covariance can be written as \( \mathbf{C}(u,v) = \sigma^2 \mathbf{R}(u - v; \theta) \) where the so-called correlation
function $R$ is a function of positive type with parameters $\theta$, and $\sigma^2$ is a scale parameter called the process variance. The choice of the covariance kernel $C$ is crucial for the construction of the kriging metamodel, all the more so when the trend is known or assumed constant (Roustant et al., 2012). Some of the most popular 1-dimensional stationary kernels include the Gaussian kernel, Fourier transform of the Gaussian density, as well as the Matern kernel, Fourier transform of the Student density (Stein, 1999). The covariance kernels available in the version of DiceKriging used in this study (Roustant et al., 2012) can be written as follow:

$$c(h): C(u,v) = \sigma^2 \prod_{j=1}^{d} g(h_j; \theta_j)$$

(20)

Where $h = (h_1, \ldots, h_d) := u - v$, and $g$ is a 1-dimensional covariance kernel. The parameters $\theta_j$, also called characteristic length-scale according to (Rasmussen and Williams, 2006), are chosen to be physically interpretable in the same unit as the corresponding variables (Roustant et al., 2012). The package proposes five equations for the covariance kernels function $g(h)$ (Gaussian, Matérn 5/2, Matérn 3/2, Exponential and Power-Exponential), according the analytical formula given by (Rasmussen and Williams, 2006). For the meta-models of this study we chosen the Matérn 5/2 expression for $g(h)$:

$$g(h) = \left(1 + \frac{\sqrt{5} |h|}{\theta} + \frac{5h^2}{3\theta^2}\right) \exp\left(-\frac{\sqrt{5} |h|}{\theta}\right)$$

(21)

Now, let us recall that the best linear unbiased predictor of $Y(x)$ based on the observations $Y(X)$ is obtained by finding $\lambda^*(x) \in R^n$ minimizing the mean squared error $MSE(x) := E\left[(Y(x) - \lambda(x))^T Y(X)\right]^2$. If $C$ is invertible, the strict convexity of MSE ensures both the existence and the uniqueness of $\lambda^*(x) \in R^n$, and the UK weights are given by

$$\lambda^*(x) = C^{-1}c(x), \text{ where } C = \left(C\left(x^{(i)}, x^{(j)}\right)\right)_{i \leq j \leq n} \text{ is the covariance matrix of } Y(X), \text{ and } c(x) = \left(C\left(x, x^{(i)}\right)\right)_{i \leq n} \text{ is the vector of covariance between } Y(x) \text{ and } Y(X).$$

Substituting both the random vector $Y(X)$ by its realization $y$ and $\lambda(x)$ by $\lambda^*(x)$ in the MSE equation, we get the so-called UK-mean prediction:

$$m_{UK}(x) = f(x)^T \hat{\beta} + c(x)^T C^{-1} \left(y - F \hat{\beta}\right)$$

(22)

Similarly, by plugging in the optimal $\lambda^*(x)$ in the expression of the MSE, one gets the so-called variance at $x$:

$$s_{UK}^2(x) = C(x,x) - c(x)^T C^{-1} c(x) + \left(f(x)^T - c(x)^T C^{-1} F\right)^T \left(F^T C^{-1} F\right)^{-1} \left(f(x)^T - c(x)^T C^{-1} F\right)$$

(23)
Where \( f(x) \) is the so-called vector of trend functions values at \( x \), \( F = \begin{pmatrix} f(x^{(1)}), \ldots, f(x^{(n)}) \end{pmatrix}^T \) is the \( n \times p \) so-called experimental matrix, and the best linear estimator of \( \beta \) under correlated residual is given by the usual formula

\[
\hat{\beta} := \left( F^T C^{-1} F \right)^{-1} F^T C^{-1} y.
\]

Basic properties of UK include similar interpolating behaviour, with a variance vanishing at the design of experiments. Furthermore, \( m_{\text{UK}}(x) \) tends to the best linear fit \( f(x)^T \hat{\beta} \) whenever the covariances \( c(x) \) vanish. Note also other interesting properties associated to the kriging meta-model, according to (Tanioka and Satake, 1996): it is flexible to any kind of functional form of the original model, by introducing less restrictive assumptions on the functional form of the simulator than a polynomial model would imply and it provides a variance estimate of the prediction, according to Eq. 23.
<table>
<thead>
<tr>
<th>Step for meta-models design</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial design database</td>
<td>Monte-Carlo sampling in the distribution of the model input parameters.</td>
</tr>
<tr>
<td>Splitting data/base in training set and validation set</td>
<td>Splitting the initial data base (Amari et al., 1997): 1) Training set (80% of initial database scenarios) 2) Validation set (20% of initial database scenarios)</td>
</tr>
<tr>
<td>Construction</td>
<td>Estimation of meta-model parameters according to equations reported in section 2.4.2.</td>
</tr>
<tr>
<td>Accuracy/Optimization</td>
<td>k-fold cross validation (Hastie et al., 2002), (Kohavi, 1995) and (Breiman and Spector, 1992).</td>
</tr>
<tr>
<td>Meta-model uncertainty</td>
<td>Residual analysis and R^2 for the validation set.</td>
</tr>
</tbody>
</table>

**Table 1:** Steps for meta-models construction and validation

<table>
<thead>
<tr>
<th>Seismic source parameter*</th>
<th>Design database</th>
<th>Western database</th>
<th>Eastern database</th>
<th>Gorringe bank</th>
<th>Horseshoe bank</th>
<th>Global database*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strike [degrees]</td>
<td>0 – 360</td>
<td>0 – 360</td>
<td>0 – 360</td>
<td>40 – 70</td>
<td>40 – 70</td>
<td>0 – 360</td>
</tr>
<tr>
<td>Longitude [degrees]</td>
<td>-18 – -7</td>
<td>-18 – -10</td>
<td>-10 – -7</td>
<td>-12 – -11</td>
<td>-10.5 – -9.5</td>
<td>-18 – -7</td>
</tr>
<tr>
<td>Latitude [degrees]</td>
<td>34 – 40</td>
<td>34 – 40</td>
<td>34 – 40</td>
<td>36.5 – 37.5</td>
<td>35.8 – 36.5</td>
<td>34 – 40</td>
</tr>
<tr>
<td>Depth [km]</td>
<td>2 – 60</td>
<td>5 – 10</td>
<td>5 – 30</td>
<td>10 – 40</td>
<td>10 – 40</td>
<td>5 – 40</td>
</tr>
<tr>
<td>Magnitude range Mw [-]</td>
<td>6.7 – 9.3</td>
<td>6.7 – 8.3</td>
<td>6.8 – 9.3</td>
<td>7.7 – 8.9</td>
<td>7.7 – 8.9</td>
<td>6.7 – 9.3</td>
</tr>
</tbody>
</table>

**Table 2:** Summary of the variation range of the seismic source input parameters for the design, the western, the eastern database and for the tsunami scenarios associated to the Gorringe bank and the Horseshoe bank (hypothesis from (Duarte et al., 2013) and (Grevemeyer et al., 2017)). *The global database is composed by the scenarios simulated with the meta-models (more then 50,000 tsunami scenarios) for the construction of the western and the eastern database. **Seismic source parameters are assumed uniformly distributed and are randomly sampled for the construction of the Global database.
<table>
<thead>
<tr>
<th>Gauge</th>
<th>Longitude [degrees]</th>
<th>Latitude [degrees]</th>
<th>Depth [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saint-Malo</td>
<td>-2.08</td>
<td>48.7</td>
<td>8</td>
</tr>
<tr>
<td>Brest</td>
<td>-4.65</td>
<td>48.26</td>
<td>28</td>
</tr>
<tr>
<td>La Rochelle</td>
<td>-1.65</td>
<td>45.93</td>
<td>40</td>
</tr>
<tr>
<td>Gastes</td>
<td>-1.27</td>
<td>44.35s</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 3: Location and water depth of the French Gauges chosen for meta-model construction

<table>
<thead>
<tr>
<th></th>
<th>Saint Malo</th>
<th>Brest</th>
<th>La Rochelle</th>
<th>Gastes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min [m]</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>µ [m]</td>
<td>0.21</td>
<td>0.47</td>
<td>0.37</td>
<td>0.30</td>
</tr>
<tr>
<td>µ + σ [m]</td>
<td>0.39</td>
<td>0.93</td>
<td>0.71</td>
<td>0.60</td>
</tr>
<tr>
<td>µ + 2σ [m]</td>
<td>0.57</td>
<td>1.38</td>
<td>1.05</td>
<td>0.90</td>
</tr>
<tr>
<td>Max [m]</td>
<td>1.60</td>
<td>5.66</td>
<td>3.12</td>
<td>3.07</td>
</tr>
</tbody>
</table>

Table 4: Maximum water height associated with the design database; µ, σ and Max correspond to the mean, standard-deviation and maximum modelled values.

<table>
<thead>
<tr>
<th>Gauges</th>
<th>Training set</th>
<th>Validation set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L_CV [-]</td>
<td>R2 [%]</td>
</tr>
<tr>
<td>Saint-Malo</td>
<td>0.004</td>
<td>87%</td>
</tr>
<tr>
<td>Brest</td>
<td>0.012</td>
<td>94%</td>
</tr>
<tr>
<td>La Rochelle</td>
<td>0.029</td>
<td>76%</td>
</tr>
<tr>
<td>Gastes</td>
<td>0.021</td>
<td>80%</td>
</tr>
</tbody>
</table>

Table 5: Summary of meta-model evaluations with the testing and the training data base; µᵣ, σᵣ and Maxᵣ correspond to the mean, standard-deviation and maximum residual respectively. L_CV is defined in Eq. 5.

<table>
<thead>
<tr>
<th>Hypothesis reported in Allgeyer et al. (2013)</th>
<th>Results from meta-models constructed in this study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>Saint Malo [m]</td>
</tr>
<tr>
<td>Johnston (1996)</td>
<td>0.37</td>
</tr>
<tr>
<td>Baptista et al. (2003)</td>
<td>0.20</td>
</tr>
<tr>
<td>Gutscher et al. (2006)</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 6: Maximum tsunamis height computed with meta-models using seismic sources simulated in (Allgeyer et al., 2013) for the 1755 Lisbon-tsunami.
Table 7: Maximum water height associated with the global database; $\mu_m$, $\sigma_m$ and $\text{Max}_m$ correspond to the mean, standard-deviation and maximum modelled values.

<table>
<thead>
<tr>
<th></th>
<th>Saint Malo</th>
<th>Brest</th>
<th>La Rochelle</th>
<th>Gastes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_m$ [m]</td>
<td>0.09</td>
<td>0.18</td>
<td>0.19</td>
<td>0.13</td>
</tr>
<tr>
<td>$\mu_m + \sigma_m$ [m]</td>
<td>0.17</td>
<td>0.34</td>
<td>0.36</td>
<td>0.27</td>
</tr>
<tr>
<td>$\mu_m + 2\sigma_m$ [m]</td>
<td>0.24</td>
<td>0.51</td>
<td>0.54</td>
<td>0.41</td>
</tr>
<tr>
<td>$\text{Max}_m$ [m]</td>
<td>0.77</td>
<td>2.01</td>
<td>1.81</td>
<td>1.92</td>
</tr>
</tbody>
</table>

Table 8: Numerical tsunami database and scenarios-based hazard levels. For each gauge, the uncertainty corresponds to the $\pm 2\sigma_r$ also reported in Table 5. The maximum modelled tsunamis height is also reported in the next column.

<table>
<thead>
<tr>
<th></th>
<th>HorseShoe ($\mu_m$) [m]</th>
<th>Gorringe ($\mu_m$) [m]</th>
<th>Global Database ($\mu_m + 2\sigma_m$) [m]</th>
<th>Meta-model uncertainty ($\pm 2\sigma_r$) [m]</th>
<th>$\text{Max}_m$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saint Malo</td>
<td>0.18</td>
<td>0.25</td>
<td>0.24</td>
<td>$\pm 0.13$</td>
<td>0.77</td>
</tr>
<tr>
<td>Brest</td>
<td>0.31</td>
<td>0.53</td>
<td>0.51</td>
<td>$\pm 0.22$</td>
<td>2.01</td>
</tr>
<tr>
<td>La Rochelle</td>
<td>0.26</td>
<td>0.32</td>
<td>0.54</td>
<td>$\pm 0.34$</td>
<td>1.81</td>
</tr>
<tr>
<td>Gastes</td>
<td>0.18</td>
<td>0.29</td>
<td>0.41</td>
<td>$\pm 0.29$</td>
<td>1.92</td>
</tr>
</tbody>
</table>

5
**Figures**

<table>
<thead>
<tr>
<th><strong>STEP 1: PROBLEM SPECIFICATION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goal:</strong> enable the modelling of maximum tsunami heights generated by a chosen tsunamigenic area</td>
</tr>
<tr>
<td><strong>Method:</strong> generation of tsunamis according to Okada (1985) surface deformation model and 2D propagation of tsunamis using Boussinesq equations as reported in Allgeyer et al. (2013)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>STEP 2: META-MODELS CONSTRUCTINO AND VALIDATION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goal:</strong> replace the numerical model, reduce the computational coast, extensive exploration of the seismic area</td>
</tr>
<tr>
<td><strong>Method:</strong> kriging meta-model (see section 2.2 and APPENDIX 1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>STEP 3: DTHA ASSESSMENT THROUGH UQ AND GSA</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goal:</strong> (i) estimation of all the possible tsunami heights generated by a seismic area and impacting the target zone; (ii) assessment of MCS_h (see sections 3.3, 4.1, 4.2 and 4.3)</td>
</tr>
<tr>
<td><strong>Goal:</strong> critical analysis of the relevance of the seismic source parameters on tsunami heights at a given location (see section 4.4)</td>
</tr>
<tr>
<td><strong>Method:</strong> Monte-Carlo simulations using kriging meta-model instead of the original model</td>
</tr>
<tr>
<td><strong>Method:</strong> Computation of Sobol Indices using the methodology proposed by Jansen et al. (1994). See section 2.3.2</td>
</tr>
</tbody>
</table>

*Figure 1:* Global methodology proposed in this study.
Figure 2: (a) Numerical domain used for the deterministic simulations performed in this study and (b) geometrical input parameters of the tsunami-code employed in the study. $d$ [km] is the depth of the seismic source which is assumed to be at the middle of the fault, $hL$ [km] is the half fault length, $W$ [km] is the half fault width, $x$-$y$-$z$ are three-dimensional axes.
Figure 3: Bathymetry covering the computational domain. Red cross hatching show location of tsunamigenic sources used for meta-models construction. The points represent the gauge locations selected for tsunami database construction along the French Atlantic Coast. Gorringe fault and Horseshoe fault are special structures at the boundary between the Eastern domain (on the right) and the Western domain (on the left).
Figure 4: (a) Absolute residuals frequency distribution and (b) cumulate residual distribution for French Gauges computed as the difference between the modelled and meta-modelled tsunamis height for the testing database. The Table in figure (a) summarize the absolute residuals associated with higher quantiles of the distribution.

<table>
<thead>
<tr>
<th>% finer than</th>
<th>Saint Malo [m]</th>
<th>Brest [m]</th>
<th>La Rochelle [m]</th>
<th>Gastes [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>75%</td>
<td>0.06</td>
<td>0.03</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>85%</td>
<td>0.08</td>
<td>0.08</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td>90%</td>
<td>0.10</td>
<td>0.13</td>
<td>0.25</td>
<td>0.20</td>
</tr>
<tr>
<td>95%</td>
<td>0.14</td>
<td>0.22</td>
<td>0.37</td>
<td>0.29</td>
</tr>
<tr>
<td>98%</td>
<td>0.18</td>
<td>0.42</td>
<td>0.52</td>
<td>0.41</td>
</tr>
<tr>
<td>100%</td>
<td>0.50</td>
<td>0.98</td>
<td>1.32</td>
<td>1.39</td>
</tr>
</tbody>
</table>
Figure 5: Comparison between the modelled and the meta-modelled maximum water height for the test database and the selected French Atlantic Gauges (blue points). The red line indicates the theoretical perfect correspondence between the original model and meta-model predictions. Grey lines indicates the 1σ confidence interval for meta-models predictions and black lines the 2σ confidence interval for meta-models predictions.
Figure 6: Convergence study for the numerical tsunami database built through meta-models (only Brest and La Rochelle gauges are represented). The mobile mean of the simulated water height is stable after around two thousands simulations. The 95% confidence bounds for means correspond to “+/- \( \frac{2 \times \sigma_{m(N)}}{\sqrt{N}} \)” according to the central limit theorem and the chose Monte-Carlo sampling method, where \( \sigma_{m(N)} \) is the mobile standard deviation.
Figure 7: Distribution of the maximum tsunami water heights for tsunamis scenarios generated by the Horseshoe bank (blue line) and the Gorringe bank (red line). Comparison with scenarios associated to the western shelf (green), the eastern shelf (grey) and the global data base (black line).
Figure 8: Location of the seismic sources associated with the proposed tsunami hazard “MCS_h” ($\mu_m + 2\sigma_m$) for the French Atlantic Gauges; the Magnitude associated with these tsunami scenarios varies between 7.5 and 9.
Figure 9: a) Magnitude distribution of earthquake events leading to the strongest (higher than $\mu_m + 2\sigma_m$) tsunami scenarios at the French Atlantic Gauges compared to the global data set; b) percentage of strongest tsunami scenarios (higher than $\mu_m + 2\sigma_m$) associated with western and eastern domains in the magnitude range $M_w [7.8 – 8.5]$, in which western and eastern tsunami scenarios overlap. # indicates the number of tsunami scenarios.
Figure 10: Tsunamis hazard ("$\mu_m$" modelled values) for the French Atlantic Gauges associated with the 1755-like tsunamis scenarios form the Horseshoe bank (blue) and the Gorringe bank (red). Comparison with the tsunami hazard ("$\mu_m+2\sigma_m$" modelled water height) resulting from uncertainty quantification (black line). The black bars represent the uncertainty related to the use of a meta-model instead of the original model (equal to +/-2$\sigma_r$).
Figure 11: Total order Sobol Index computed for the French Atlantic Gauges.
References


Okada, Y.: Surface deformation due to shear and tensile faults in a half-space, Bulletin of the seismological society of America, 75, 1135-1154, 1985.


Roustant, O., Ginsbourger, D., and Deville, Y.: DiceKriging, DiceOptim: Two R packages for the analysis of computer experiments by kriging-based metamodelling and optimization, 2012.


