Thanks for the comments. The following is my reply.

Reply for RC1

General comments:
The paper presents the results of a study aimed at improving a data assimilation (DA) algorithm based on the residual resampling particle filtering. Two applications are provided in order to respectively test the feasibility of the improved algorithm and show an example concerning slope instabilities. In this latter regard, a ‘synthetic’ case is presented starting from the expression of the factor of safety implemented in the TRIGRS physically-based model. From this point of view, in the abstract the positive effects of the proposed DA algorithm in the use of TRIGRS should be enhanced and, more in general, the main goal to be pursued with reference to slope stability processes should be more clearly stated. Indeed, the submitted version of the paper does not allow understanding the benefits deriving from the adoption of the improved algorithm in addressing practical issues about landslides. In my opinion, for the readers of NHESS International Journal, the paper could be of interest only if the theoretical approach is applied to a real (not to a synthetic) case study. Finally, the paper is poorly written and, in some parts, difficult to understand; in this regard, the manuscript needs some English language editing.

Reply:
In this paper, a synthetic experiment is presented to verify the feasibility of the algorithm and its application to the TRIGRS landslide model. The main goal of this study is to propose a new method and prove it can be applied to the evaluation of FS in landslide slope. Experiments of real cases is carrying out and it need some more monitoring data. Then the paper is modified in some poorly expressed places to improve the expression of English languages.

Specific comments:
Introduction – page 1, line 17. Why the (only) landslide event occurred in China on June 24, 2017 is mentioned? Section 1, Introduction – page 1, lines from 19 to 23. Considering the scope of the paper, why the authors mentioned some numerical methods for landslide modelling? And what type of landslides the authors are taking into account? The description of the TRIGRS model is very poor and should be improved. Section 1, Introduction – page 2, lines from 20 to 22. As mentioned in the general comments, the manuscript includes some sentences that appear meaningless. For example, the authors claim that they choose a ‘slope movement model’ (?) with a 10*10 size grid (no information about the dimensions are provided), applying the assimilation algorithm and TRIGRS program to ‘predict and improve the prediction’ (?) of safety factors (more than one?) and deformations (TRIGRS does not allow studying deformations) of the landslide (which?). Section 4, Application to landslide simulation based on TRIGRS model – page 6, lines from 1 to 5. Bearing in mind that TRIGRS allows simulating only the triggering stage of landslides, why the authors considered the post-failure stage? And, once again, what type of rainfall-induced landslide are they referring to? Or, more in general, what kind of physical process are they simulating and how the variation with time of the
groundwater pressure head is estimated? Section 4, Application to landslide simulation based on TRIGRS model – page 6, lines from 11 to 12. Could the authors clarify the meaning of Figure 5? Numbers in Figure are representative of what? And color shadings?

Reply:
Some extra content has been deleted, such as the landslide event occurred in China on June 24, 2017. In section 1, some methods for landslide modeling are mentioned to introduce the research status of landslide deformation analysis and numerical landslide evaluation. This study is applied to “peristaltic landslides”, which is added in the last paragraph of section 1. The description of the TRIGRS model is enriched in the beginning of section 4. In the manuscript, poor expressed contents mentioned in the comment have been modified. In section 4, the useless content of post failure stage has been deleted. To estimate the groundwater pressure head (\(\varphi\)), some content of \(\varphi\)-estimation is added to the manuscript. Formula (21) and its context is the calculation method of \(\varphi\)-estimation, and Figure 6 and Figure 7 are its change of overall distribution and single cell, respectively. The illustration of Figure 5 has been revised to “Model results and assimilation results of FS. The maps in the first row are the model results running for 5, 10, 15, 20 days respectively, and that in the second row are the assimilation results. The horizontal and vertical coordinates in each graph are grid numbers of each cell.”

Technical corrections
Thanks for your review. The manuscript has been revised.

Reply for RC2

Questions reply:
1. Extensive editing of English language and style required: this must be reviewed in depth.
   Reply: The manuscript has been revised. The text is modified in some poorly expressed places to improve the expression of English languages.
2. The improvements such as the accuracy and computation burden of the particle filter should be more clarified.
   Reply: At the end of section 2, the root mean square difference (RMSD) has been added as a measure factor to evaluate the accuracy. The main computation burden of the particle filter is explained in Para.2 of Sec.2: “Residual resample is a way to solve the problem of particle degeneracy which is an unavoidable trouble in standard PF. With the recursive progress, the weights of particles are gradually concentrated on a few samples and others tend to be zero. To keep most particles effective, low-weight particles are removed and high-weight particles are duplicated. This causes that the particle sets can hardly represent the prior PDF due to the declining of particles diversity.”
3. Section 4, the authors mentioned “observations are generated from the Fs by adding a disturbance with normal distribution \(N(0.2, 0.3)\)”, why the mean of disturbances is 0.2 rather than 0?
Reply: Due to the TRIGRS model calculate the safe factor cell by cell, without considering the interaction force between grid cells, the TRIGRS output results have systematic errors. So, we assumed a disturbance with an experience mean of 0.2.

4. I noticed that the FS was chosen as the assimilated factor, why not use the displacement?

Reply: In the post failure stage of landslide, the two variables, FS and displacement (in fact the integration of displacement velocity over time, \( dv/dt \)), can be converted to each other. The FS determines the integration of displacement velocity over time. When the displacement is chosen as the assimilated factor, it is necessary to convert the FS to velocity, and then accumulate to get displacement by time. This progress would magnify the error of FS, and the difference between model value of displacement and the observation would be larger. That would reduce the efficiency of particle filter. To convert the displacement to FS can control the dispersion of errors. Besides, this also reduces computational complexity. Therefore, FS is more suitable to be the assimilated factor than displacement.

5. Data assimilation is usually applied on large scale scenarios. This study employed assimilation size 10*10, I suggest you increase the assimilation size, or use true landslide monitoring data instead.

Reply: In the 3rd paragraph of section 4, the size of the assimilation area has been increased. “An example of 10 * 10 grid TRIGRS model is set to be the background, and each grid cell is a square with a length of 10 meters.”

In this paper, a synthetic experiment is presented to verify the feasibility of the algorithm and its application to the TRIGRS landslide model. The main goal of this study is to propose a new method and prove it can be applied to the evaluation of FS in landslide slope. Experiments of real cases is carrying out and it need some more monitoring data.

Comments reply:
1. The full name of “TRIGRS” should be given at its first appearance.

Reply: Thanks. The full name of “TRIGRS” is added in the first paragraph of introduction.

2. Page 1 Line 7 and 8, I think it would be better to recognize this sentence.

Reply: The manuscript has been modified to

“In this work, an improved particle filter algorithm is proposed. To overcome the particle degeneration and improve particles’ efficiency, the processes of particle resample and particle transferring are updated.”

3. Page 1 Line 23, reference missing: ‘Jiang adopted the Ensemble Kalman filter to landslide movement model in relation to hydrological factors, which introduce data assimilation (DA) to landslide.’

Reply: The reference has been added.


4. Page 2 Line 14: ‘It can get good results to using...’ should be ‘to use’.

Reply: Thanks. The manuscript has been modified. Some other expression errors have also been modified.
Data Assimilation with an Improved Particle Filter and its Application in the TRIGRS Landslide Model

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Abstract. Particle filter has become a popular algorithm in data assimilation for its ability to handle non-linear or non-Gaussian state-space models, while still being seriously influenced by its disadvantages. In this work, the particle filter algorithm is improved, proposing two methods to overcome the particle degeneration and improve particles' efficiency. In this algorithm, particle propagating and resample method are ameliorated. An improved particle filter algorithm is proposed. To overcome the particle degeneration and improve particles' efficiency, the processes of particle resample and particle transferring are updated. In this improved algorithm, particle-propagation and the resampling method are ameliorated. The new particle filter is applied to the Lorenz-63 model, verified and its feasibility and effectiveness are verified using only 20 particles. The root mean square difference (RMSD) of estimation converges to stable when there are more than 20 particles. Finally, we choose a 10 x 10 grid slope model of TRIGRS a peristaltic landslide model and carry out an assimilation experiment. Results show that the estimations of states can effectively correct the running-offset of the model and the RMSD is convergent after 3 days of assimilation.

Key words. Data assimilation, particle filter, nonlinear model, Lorenz-63, TRIGRS landslide model

1 Introduction

A mount of Mountainous areas all over the world have suffer frequent landslide disasters all over the world. People living in mountainous areas are faced with the threat of landslide disasters. A destructive landslide occurred on June 24, 2017 located at 103°39' 03"E, 32°04' 09"N, Maoxian County, Sichuan province, China, caused a huge loss of personal and property. Works of landslide monitoring, analysis and forecasting are crucial. Many numerical modeling methods of slope evolution have been proposed and developed recently, such as discontinuous deformation analysis (DDA) (Shi 1992, Jing, Ma et al. 2001, Ma, Kaneko et al. 2011) and distinct elements methods (DEM) (Lorig and Hobbs 1990, Marcato, Fujisawa et al. 2007, Li, He et al. 2012). Iverson carried out the TRIGRS program to predict the stability of landslides in response to rainfall. It is a raster-based model, depends on time for transient rainfall infiltration (Iverson 2000, Baum 2008). Iverson proposed a mathematical
model that uses Richards’ equation to evaluate effects of landslides in response to rainfall infiltration (Iverson 2000). The Transient Rainfall Infiltration and Grid-based Regional Slope-stability (TRIGRS) model is a raster-based model, and depends on time of transient rainfall infiltration (Baum, Savage et al. 2008). Jiang adopted the Ensemble Kalman filter to landslide movement model in relation to hydrological factors, which introduced data assimilation (DA) to landslide (Jiang, Liao et al. 2016).

Data assimilation is a common approach to solve an estimation of optimal states in dynamic systems. With DA algorithms and operators, DA merges different scales of observations into dynamic models to take advantage of all the information. Many DA algorithms have been developed and improved in recent years, in which particle filter (PF) is a popular algorithm for its ability to handle availability under conditions of nonlinear and non-Gaussian distributed models (Arulampalam, Maskell et al. 2002, Moradkhani, Hsu et al. 2005). The increasing applications and improvements of PF have been researched recently in DA and other fields.

Salamon, et al. (Salamon and Feyen 2009) applied the residual resampling particle filter (RRPF) to assess parameter, precipitation, and predictive uncertainty in rainfall–runoff model. Thirel, et al. (Thirel, Salamon et al. 2013) assimilated the snow-covered areas in physical distributed hydrological models and MODIS satellite data to improve the pan-European flood forecasts. Mattern, et al. (Mattern, Dowd et al. 2013) carried out assimilation experiments for a three-dimensional biological ocean model and satellite observations and verified the feasibility of biological state estimation with sequential importance resampling (SIR) for realistic models.

However, large computational complexity and particle degradation or collapse are still obstacles in PF. To solve these problems, some resampling algorithms have been proposed. One improvement is adding an item related to observations, to make the proposal density dependent on the future observations, accordingly most particles could situate into the range of observation error (van Leeuwen 2010). This method can achieve good results using only 10~20 particles in high-dimensional assimilation experiments. But the number of key particles are reduced when the system variance is larger than the observed variance, and the values of added items are uncertain. Another improvement is to replace the duplicated process by generating a Halton sequence in residual resampling (Zhang, Qin et al. 2013). The disordered particle sets are turned into ordered sets and too few particles can hardly describe the posterior probability density function (PDF) better.

In this paper, a new resampling approach is proposed to improve the above method, maintaining both particles’ diversity and efficiency. Applying to Lorenz-63 model using different numbers of particles range from 10~200, this method has shown its efficiency and sensitivity to the number of particles. Finally, we choose a slope movement model with a 10*10 size grid, applying the assimilation algorithm and TRIGRS program to predict and improve the prediction of safety factors and deformations of the landslide.

In section 2, a new resampling approach is proposed to improve the above method, maintaining both particle diversity and efficiency. The new algorithm formula and implementation process are listed in the text. To evaluate the safety factor of peristaltic landslide in slow deformation process, a simulation experiment, applied to Lorenz-63 model using different numbers of particles, ranging between 10~200, is explained in section 3, which demonstrates that the new method shows efficiency and
sensitivity to the number of particles. Finally, a rainfall infiltration landslide model case is analyzed. We choose an experimental landslide model with a 10 * 10 grid as background to conduct an assimilation experiment. The improved assimilation algorithm is applied to TRIGRS program to evaluate the change of factor of safety (FS) in the experimental model.

2 Improvements of Residual Resampling Particle Filtering

In sequential importance sampling, the state vector is represented by a set of particles

$$x_k = f(x_{k-1}) + G_k(x_{k-1})\varepsilon_k$$  \hspace{1cm} (1)

where $x$ is the state vector with initial PDF $p(x_0)$, $k$ is the subscript of time steps, $\varepsilon_{k-1}$ is system noise with zero mean at step $k-1$, and $f(\cdot)$ is the model operator. Initial $N$ particles are sampled from $p(x_0)$. The observation equation is

$$z_k = h(x_k) + \eta_k$$  \hspace{1cm} (2)

where $z$ is the observation vector, and $h(\cdot)$ is the observation operator. Weights of particles are calculated by (3), and normalized to get $w_k^i$

$$\tilde{w}_k^i = w_{k-1}^i \cdot \frac{p(z_k | x_k^i) p(x_k^i | x_{k-1}^i)}{q(x_k^i | x_{k-1}^i, z_k)}$$  \hspace{1cm} (3)

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{j=1}^{N} \tilde{w}_k^j}$$  \hspace{1cm} (4)

where $p(z_k | x_k^i)$ is the likelihood of observation, and $q(x_k^i | x_{k-1}^i, z_k)$ is the proposal function.

Residual resample-resampling is a way to solve the problem of particle degeneracy which is an unavoidable trouble in standard PF. With the recursive progress, the weights of particles are gradually concentrated on a few samples and others are tend to be zero. To keep most particles effective, low-weight particles are removed and high-weight particles are duplicated. This causes that the particle sets can hardly represent the prior PDF due to the declining of particles diversity. Some improvements about the residual resample-resampling algorithm are proposed in this paper. Firstly, in the process of particle transferring, we choose

$$x_k^i = f(x_{k-1}^i) + \hat{\varepsilon}_{k-1} + J_k[z_k - h(\hat{x}_{k-1})]$$  \hspace{1cm} (5)

where $J_k$ is a coefficient like the “gain” in an extended Kalman filter:

$$J_k = D_{k/k-1} B_k^T [B_k D_{k/k-1} B_k^T + R_k]^{-1}$$

$$D_{k/k-1} = A_{k-1} D_{k-1/k-1} A_{k-1}^T + G_{k-1} (\hat{x}_{k-1}) Q_{k-1} G_{k-1}^T (\hat{x}_{k-1})$$  \hspace{1cm} (6)

in which $A_k$, $B_k$ are the linearization parameter of $f(\cdot)$ and $h(\cdot)$, respectively:
\[ A_k = \frac{\partial f_k}{\partial x_k}(\hat{x}_k), \quad B_k = \frac{\partial f_k}{\partial x_k}(\hat{x}_{k-1}) \]  

(7)

\( D_{k/k} \) is estimation variance of state \( x_k \) at step \( k \). This process is equal to translating particles close to observations. But the value of \( J_k \) is hard to determine because the variance of state estimation \( D_{k-1/k-1} \) in PF is difficult to compute.

To simplify the calculation, suppose that the translated particles are a series of virtual observations about the state at step \( k \).

Write the particle set as:

\[ X_{k/k}^N = \{ x_{k/k}^i \}_{i=1,2,...,N} \]  

(8)

and replace \( D_{k-1/k-1} \) with the variance of particles. To keep the value of \( D_{k-1/k-1} \) unchanged before and after translation, we choose the posterior particles at step \( k-1 \):

\[ D_{k-1/k-1} = \text{var}(X_{k-1/k-1}) \]  

(9)

Secondly, using the method of Zhang et al. (Zhang, Qin et al. 2013) to compute accumulative copy times (ACT), each parent particle with high weights regenerates a set of new particles. Differently, instead of duplicating or generating Halton sequence, it generates a series of normally-distributed particles:

\[ \{ x_k^1, x_k^2, ..., x_k^{ACT} \} \sim N(x_k^j, G_k(x_k^j)) \]

where \( ACT \) is the ACT of the \( i \)th particle, and the mean and variance are related on the value of the parent. Accordingly, the resampled particle set is composed of some different particle sets which obey normal distribution. Assume that the \( j \)th progeny particle of \( x_k^i \) is written as \( x_{k}^{i,j} \), the formula (3) can be written as:

\[ \bar{w}_k^j = w_k^i, \quad p(z_k | x_k^j) p(x_k^j | x_{k-1}) \]  

\[ q(x_k^j | x_{k-1}, z_k) \]  

(10)

**Shortly Briefly**, the improved RRPF in this section can be implemented by the following steps:

**Step 1:** Draw initial particles \( \{ x_0^i \} \) from prior PDF \( p(x_0) \).

**Step 2:** Compute the mean and variance of posterior particles at step \( k-1 \):

\[ \bar{x}_{k-1/k-1} = \frac{1}{N} \sum_{i=1}^{N} x_{k-1/k-1}^i \]  

(11)

\[ D_{k-1/k-1} = \frac{1}{N-1} \sum_{i=1}^{N} (x_{k-1/k-1}^i - \bar{x}_{k-1/k-1})(x_{k-1/k-1}^i - \bar{x}_{k-1/k-1})^T \]  

(12)

**Step 3:** Using the new method in this section, compute the “gains” of particles:

\[ D_{k/k} = \begin{bmatrix} \frac{\partial f_k}{\partial x_k}(\hat{x}_k) \\ \frac{\partial f_k}{\partial x_k}(\hat{x}_{k-1}) \end{bmatrix} D_{k-1/k-1} \begin{bmatrix} \frac{\partial f_k}{\partial x_k}(\hat{x}_k) \\ \frac{\partial f_k}{\partial x_k}(\hat{x}_{k-1}) \end{bmatrix}^T + G_{k-1}(\hat{x}_{k-1})Q_{k-1}G_{k-1}^T(\hat{x}_{k-1}) \]  

(13)
\[ J_k = D_{k/k-1} \left[ \frac{\partial f_k}{\partial x_k} (\hat{x}_{k/k-1}) \right] \left[ \frac{\partial f_k}{\partial x_k} (\hat{x}_{k/k-1}) \right]^T + R_k \]  

\[ (14) \]

Step 4: Transfer the particles close to the observation:

\[ x_i' = f(x_i') + \hat{e}_{k-1} + J_k [z_k - h(\hat{x}_{k-1})] \]

\[ (15) \]

Step 5: Residual resampling. Each particle generates a set of normal-distributed progeny particles, and all progeny sets make up the resampled particle set:

\[ \{ x_1^{iACT}, x_2^{iACT}, ..., x_i^{ACT} \} = N(x_i', G(x_i')) \]

\[ (16) \]

\[ \{ x_1^{ACT}, x_2^{ACT}, ..., x_N^{ACT} \} = \{ x_i^n \} \]

\[ (17) \]

When \( ACT_i = 0 \), \( x_i^{iACT} \) is an empty set.

Step 6: Compute and normalize weights:

\[ \tilde{w}_k = w_{k-1} \cdot p(z_k \mid x_i') \]

\[ (18) \]

\[ w_k^i = \frac{\tilde{w}_k^i}{\sum_{j=1}^{N} \tilde{w}_k^j} \]

\[ (19) \]

Step 7: Compute the state estimation:

\[ \hat{x}_{k/k} = \sum_{i=1}^{N} x_i^i \cdot w_k^i \]

\[ (20) \]

A measure to assess the accuracy of calculation is the root mean square difference (RMSD), which is defined as

\[ RMSD = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\hat{X}_t - X_{t}^{obs})^2} \]

\[ (21) \]

where \( T \) is the period of assimilation, \( \hat{X}_t \) and \( X_{t}^{obs} \) are the assimilated value and the observation of state at time \( t \), respectively.

### 3 Application to Lorenz-63 model

We choose the Lorenz-63 model as an example to test the improved algorithm (Baines 2008).

\[ \frac{dx}{dt} = \sigma(y - x) \]

\[ \frac{dy}{dt} = x(\rho - z) - y \]

\[ \frac{dz}{dt} = xy - \beta z \]

\[ (22) \]
where the constants $\sigma$, $\rho$ and $\beta$ are system parameters proportional to the Prandtl number, Rayleigh number, and certain physical dimensions of the layer itself. Parameters are given by: $dt = 0.01$, $\sigma = 10$, $\rho = 28$, $\beta = 8/3$, the observation error $\sigma_{obs} = \sqrt{2}$, model transmission error based on time interval $\sigma_{mod} = 2\Delta t$. Initialize the filter with the starting point which is set to $(x_0, y_0, z_0) = (1.50887, -1.531271, 25.46091)$. The truth is obtained by the formula of the model recursively.

Observations are generated from the truth by adding a disturbance every 40 steps. Recurs, with 1000 recurring steps, and assimilate assimilating the observation with the model when observation exists at current step and recurs to next step when there is no observation.

Figure 1 shows the results of $x$-component using new PF with 20 particles. Note that the new PF procedure is close to the truth with much fewer particles which is more efficient than the standard PF procedure with hundreds of particles. Compute the confidence interval with 95% level using the posterior particles every step. Figure 2 shows that the intervals contain observations at almost all the steps with where observations exist. That means particle sets after translation are closed closer to observations and true states. The evolution of all particles is displayed in Figure 3, in which most particles are very close to observations except for several ones at moments with when the state changed obviously. Consider the root mean square difference (RMSD) of the estimation with respect to particle numbers as the following formula

$$\text{RMSD} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (\hat{X}_t - X_{t, obs})^2}$$

where $T$ is the period of assimilation, and $X_{t, obs}$ are the assimilated value and the observation of state at time $t$. The RMSE sequence is shown in Figure 4. The RMSD sequence is shown in Figure 4, it tends to be stable when the number of particles is more than 20. This means the improved algorithm only needs no less than 20 particles.

4 Application to landslide simulation based on TRIGRS model

TRIGRS is a program modelling rainfall infiltration, using analytical solutions for partial differential equations which represents one-dimensional, vertical flow in isotropic, homogeneous materials for simply saturated or unsaturated conditions. It computes changes of rainfall pore-pressure and factor of safety (FS) with rainfall infiltration. The FS is computed using a simple infinite-slope model cell-by-cell.

The factor of safety (FS) in TRIGRS is calculated as follows:

$$FS = \frac{\tan \phi}{\tan \alpha} + \frac{c - \varphi(Z,t)\gamma_w \tan \phi}{\gamma_s Z \sin \alpha \cos \alpha}$$

(23)
in which \( c \) is soil cohesion, \( \alpha \) is slope angle, \( \phi \) is soil friction angle, \( \varphi \) is the ground-water pressure head depending on depth \( Z \) and time \( t \), \( \gamma_w \) is ground-water unit weight and \( \gamma_s \) is soil unit weight at saturation.

The equation of post-failure motion depending on time is
\[
\frac{1}{g} \frac{dv}{dt} = -\sin \alpha [1 - F_s(Z, t)]
\]
where \( g \) is gravitational acceleration, \( v \) is downslope landslide velocity.

An example of 10 * 10 grid TRIGRS model is set to be the background, and each grid cell is a square with a length of 10 meters. The simulated observations are generated from the \( F_s \) by adding a disturbance with normal distribution \( N(0.2, 0.3) \). Due to the difficulties of determining the parameter \( \varphi \), the groundwater pressure head, and its highly sensitivity to results, we now generate a set of particles \( \{ \varphi^i \}_{k} \) form \( \varphi \), in which \( k \), and \( i \) are indices of step and particle number, respectively. The input model variance of \( \varphi \) is 2 and observation variance of \( F_s \) is 0.3. At each step, \( \varphi \) and \( F_s \) will be updated, and the updated parameters continue to participate in the next step operation as initial parameters. The number of particles is set to 20 in the particle filter program. Figure 5 shows the model-running results and the assimilated results of FS running for 5 days, 10 days, 15 days and 20 days, respectively. In the model-running results, the value of FS is smaller and decreases rapidly, while in the assimilated results the change is relatively gentle.

To evaluate the distribution variation of \( \varphi \), we propose that the estimation of \( \varphi \) is calculated as formula (
\[
\hat{\varphi}_{k+1} = \sum_{i=1}^{N} \varphi^i \cdot w_k^i
\]

in which \( w_k^i \) is calculated using formula (18) and (19). Actually, the estimation of \( \varphi \) uses the same method and particles of the estimation of \( F_s \). Figure 6 shows the distribution variation of \( \varphi \) running for 5 days, 10 days, 15 days and 20 days, respectively. The change of \( \varphi \) estimation in a single cell is illustrated in Figure 7, considering the middle unit, grid cell (5, 5).

The root mean square difference of the whole grid of points is calculated to measure the estimated error as follow.

To assess estimations of all grid cells, the root mean square difference of the whole grid of points to measure the estimated error is modified to
\[
RMSD_{grid} = \sqrt{\frac{1}{N_p} \sum_{i,j} (\hat{X}_{ij} - X_{ij}^{obs})^2}
\]
where \( N_p \) is the total number of grid points, \( i, j \) are the indices of the row and column number respectively. The RMSD curve with assimilating days is shown in Figure 8 which suggests the value is large in the first 2 days of initialization, fluctuating in next days and steady when there are no observations.
5 Conclusion and discussion

The problems of particle degeneration and efficient expression of posterior PDF are long-term difficulties which affect the
performance of particle filter. Many resampling methods can improve effectiveness of particles, but they still need a large
number of samples resulting in a large amount of computation.

In this study, we propose two approaches to improve the particle filter process. Firstly, for the problem of particle degeneration,
new Gaussian-distributed offspring particles are generated for each mother particle. This avoids particle duplication and
maintains particle diversity. Secondly, in order to improve the propagating efficiency of a priori particle into a posteriori
particle, an additional item is added which is similar to the Kalman gain at the step of particle propagation, which greatly
reduces the number of particles required. It uses only dozens of particles to achieve good results. A simulating experiment
of the Lorenz-63 model is carried out to validate the feasibility of these methods. The TRIGRS landslides model is firstly
proposed to apply to the assimilation system. Results show that the assimilating process can make the estimation close to
observations, which proves the availability-feasibility of applying the improved particle filter to the landslide model.
However, some disadvantages are still present. Grid cells are independent of each other in TRIGRS, and this leads to the FS
estimations being greater than the actual values. Therefore, the FS estimations only provide a reference for the actual values. The experiment needs improvement.

Acknowledgments. This work is financially supported by the National Key Basic Research Program of China (Grant No.
2013CB733205).
References


Figures

Figure 1: Results of new PF for the Lorenz-63 model of \(x\)-component. The red crosses are observations, the black line is the true state and the blue line is represents the new PF results.
Figure 2. The 95% confidence interval computed by posterior particles. The green dashed lines denote the upper and lower limits of the interval and the red crosses are observations.
Figure 3. The evolution of posterior particles in time. The green dashed lines show the traces of all particles, the red crosses denote the observations.
Figure 4. RMSD of the estimation with respect to particle numbers. The value is relatively high when the particle number is less than 20, and tend to be stable when more than 20.
Figure 5. Model results and assimilation results of FS. The maps in the first row are the model results running for 5, 10, 15 and 20 days respectively, and that in the second row are the assimilation results. The horizontal and vertical coordinates in each graph are grid numbers of each cell.
Figure 6. The distribution variation of groundwater pressure head (ϕ) with assimilated time. The horizontal and vertical coordinates in each graph are the grid numbers of each cell.
Figure 7. The changing line of the groundwater pressure head ($\psi$) estimation of grid cell (5, 5) with assimilating time. The value is growing with the evolution of the landslide.
Figure 8. RMSD line of all grids depending on assimilating time. The TRIGRS model is assimilated with observations in the first 20 days, and results of 21~30 days are model-running results without observations assimilated.