Characterization agricultural vulnerability to drought in the Northeast of Brazil

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Abstract. The main objective was to create an indicator of agricultural vulnerability to drought in the Northeast of Brazil (NEB). The data used for precipitation belong to ANA (Agência Nacional das Águas) considering the climatological norm from 1979-2008. Data on agricultural productivity and demographic characteristics were obtained in the agricultural census of IBGE (Instituto Brasileiro de Geografia e Estatística) in 2006 and, finally, data on natural disasters in the period 1991-2010 with CEPED (Centro de Estudos e Pesquisa em Engenharia e Defesa Civil). The Multivariate Statistical Analysis Factorial technique allowed to reduce the number of variables and to estimate a model of the sensitivity component that reproduced 42% of the original variance, besides the factors trying to represent the productive dynamics of the NEB. The results show that the Southern NEB presented the highest degree of agricultural vulnerability (17.81-121.44) in the 2000 census, when compared to the census of 2010. In the Southwest it is observed that a part of the semi-arid region presented a moderate degree (0.74-1.08) and much higher in extension when compared to the 2000 census, evidencing that exposure to drought does not directly influence the agricultural sensitivity in the most productive areas of the region. The adaptive capacity factor presented significant results for the composition of the indicator of agricultural vulnerability, mainly in the semi-arid region that varied from (0.71-5.42). In this way, it is concluded that, between the census, the southern and central part of the NEB reduced agricultural vulnerability, but the region should benefit from early warning systems as well as the development and adoption of natural resources and technology management, with the objective of educating producers about the potential impacts of extreme events.

1 Introduction

The Brazilian northeast region (NEB) is considered the most vulnerable region, due to climatic variations such as irregularity of rainfall, water deficit and low social and economic indicators (Torres et al., 2012). The high temporal and spatial variability of rainfall in this region encourages some studies to seek for a characterization of extreme precipitation events, for example, the work of Oliveira et al. (2014) affirmed an increase of scale and seasonality of precipitation in the autumn months. On the
other hand, Rao et al. (2015) determined that in the central area of the NEB, the rainy season occurs from December to May, while on the East it is from March to July.

A large part of the rural population in the semiarid regions of the planet lives under some vulnerability degree, whether by socioeconomic or climatic reasons (França et al. 2012). Also, the climate change along these last decades have caused damage in various economic activities, particularly in agriculture (Silva & Lucio, 2014). According to the Intergovernmental Panel on Climate Change-IPCC (2014), agriculture is the economic activity that will be more affected by climate change in the world, especially in developing countries.

According to the IPCC, in their fifth evaluation report AR5, the vulnerability is defined as a tendency or predisposition to a system to be adversely affected. Those are a variety of concepts and elements, including sensitivity or susceptibility to any injury caused by a stressful event and the lack of capacity to deal with it and to adapt. Therefore, the concept of agricultural vulnerability is present in some countries, such as Mexico (Luers et al., 2003); (Sánchez-Cortés & Chavero, 2010); Ghana (Antwi-Agyei et al., 2012); China (Simelton et al., 2009). In addition, it is shared in science sectors like: social (Ahmed et al., 2009); economics (Ibarrarán, Ruth, Ahmad, & London, 2007); Health (Cockroach & Confalonieri, 2011) and climate (Brooks et al., 2005); (Karim & Mimura, 2008).

The identification of a population vulnerability to climate, or specifically agricultural drought, requires large amount of data with good spatial and temporal resolution, as well as the use of forecast models for harvests or climate (Popova et Al. 2014). In Bulgaria, Alexandrov and Hoogenboom (2000) used statistical techniques to relate climate projections of precipitation and temperature with the yield of crops of maize and wheat. These authors used general circulation models (GCM) to create the climatic scenarios and assess the impact on crop yield.

Despite its frequent use in recent years, the concept of vulnerability is rarely cast in analytical measurements that could be used to advise intervention policies and assess the impacts. Therefore, the demand for researches that prioritize adaptation policy have greater importance in society due to the frequency of extreme weather patterns (Luers et al., 2003; Nelson et al., 2010)

Besides the elaboration of mitigating measures, there is the development of preparation plan that includes forecasting, monitoring, assessing the vulnerability of sectors and regions, and assistance to respond the impacts of drought should be proposed.

Given the context presented previously, the main objective of this study was to diagnose areas of agricultural vulnerability in NEB, specifically to calculate an indicator of agricultural vulnerability, using precipitation data and agricultural productivity data of various cultures as a tool for the investigation of impacts due to the occurrence of drought events in this region.

In item 2, the methodology and the study area will be described, the calculation of the climatic risk that used the SPI dry indicator and dry records from 1991 to 2010, the indicator of agricultural sensitivity with productivity data of various agricultural crops and in the period of 1990 to 2010. Finally, the indicator of adaptation capacity, by means of data from the Ministry of Social Integration. The results and discussion of the calculated indicators are presented in item 3. Moreover, to conclude, item 4 deals with the conclusions of the study.
2 Methodology

2.1 Study area

The NEB is formed by nine states of the Brazilian federation, comprising an area of approximately 1.6 million km² (IBGE, 2011). Located in the equatorial range, it presents a precipitation variability typical of these regions. According to Alvarens et al. (2013), the NEB has mostly two climates, tropical and semi-arid. The tropical climate is classified in Af (without dry season), Aw (with dry winter) and As (with dry summer). Also, the semi-arid climate has the caatinga endemic biome, that is present in all states of NEB and the largest portions are in the states of Rio Grande do Norte (61.2%) and Pernambuco (61.7%). The total annual precipitations in this region is below 700 mm, in addition it presents high average. The Figure 1 shows the study area and the precipitation stations used (red dots).

Figure 1

2.2 Data

Precipitation data were provided by the National Water Agency (ANA), and were analyzed from January 1st of 1979 to December 31st of 2008. This same database was used for the studies of Oliveira et al., (2012; 2014). The data associated to the drought disaster were catalogued by the Center for Studies and Research in Civil Engineering and Defense (CEPED) in the period from 1991 to 2012. The data on agricultural productivity and irrigation were obtained with the Brazilian Institute of Geography and Statistics (IBGE) concerning the 2006 census and the numbers for cisterns built in the region were used from the federal project in combating drought, through the semi-arid articulation (ASA), where the values go up to the year 2012. The Table 1 displays the source for each data set obtained.

Table 1.

2.3 Methods

The vulnerability function was the same proposed by Kienberger et al. (2009) described in the following equation:

\[ V = f(H, S, AC) \]  

Where: H is the danger factor, S is the sensitivity and AC is the adaptive capacity.

In Equation 1, the definition of vulnerability is measured by risk or danger (H) to a particular physical event that a society or community is exposed to. The sensitivity or susceptibility (S), which determines the intensity that the system is affected, in a positive or negative way before the stressor event and the ability to adapt (AC) that is the response of how the community is able to face these events (mitigation).

The H-risk factor was calculated according to the IPCC AR5, as described in Equation 2:

\[ H = p(E)^{n} \]
Where \( p(E) \) is the probability to happen an extreme event and \( n \) is the number of people affected by this event.

According to Kienberger et al. (2009), the first term of Equation 1 can be very difficult to measure due to the scarcity of biophysical and socioeconomic data in poor areas, however it can be written according to a specific hazard (drought, flood, erosion areas etc.). In that research, unlike the author, the risk factor was determined using the SPI proposed by (Mackee, et al, 1993) and the decrees of drought by county. In this way, the risk is directly linked to the frequency of the event, magnitude and the population affected by the drought.

So equation 2 can be rewritten as follows:

\[
H_{\text{drought}} = f(\text{SPI}, D) \Rightarrow f([\exp(MD) \times \log(D)])
\] (3)

The SPI is the drought-standardized index was calculated using monthly precipitation data provided by ANA, with a temporal range from January 1st of 1979 to December 31th of 2011. The MD is the drought magnitude and D is the disaster average per micro-region. The drought magnitude (MD) was determined in the rainy period, according to the equation proposed by Mckee et al. (1993):

\[
MD = - \sum_{j=1}^{n} SPI_{ij}
\] (4)

Where \( j \) corresponds to the month of the rainy period start that goes up to month \( n \), and \( i \) represents the year of the time series.

Additionally, D is the average number per micro-region of drought disasters decreed in the period from 1991 to 2012, published by CEPED.

The statistical analysis was performed using the software R (R Core Team, 2013). For the creation of the agricultural productivity sensitivity index (SeA), it factorial analysis technique was applied to the data set that contained information of the productive characteristics: crops (temporary and permanent), extractivism (Plant and animal), defined by the IBGE. The agricultural productivity data period was from 1990 to 2010, so the data set was divided into two sampling periods, P1 (1990 to 1999) and P2 (2000 to 2010).

This technique is widely used in studies to determine the vulnerability in various areas of knowledge such as: Climatic Vulnerability (Ford et al., 2006; Confalonieri, 2007; Barta & Confalonieri, 2011); Agricultural Vulnerability (Luers et al., 2003; O'Brien et al., 2004). The main objective of the factorial analysis is to reduce the number of variables and build, based on the estimated factors, new variables with a degree of variability relative close to the original variables. This is important to identify what characteristics are really needed in defining vulnerability to climatic extremes and which productive areas can be influenced by these changes. The equations that comprise the factorial model and estimation of the commodities are in annex A.

In relation to the determination of the adaptability capacity factor (AC), the number of establishments that used irrigation system \((N_i)\) were considered. This factor is interesting because it is connected directly to the availability of natural resources and it is inserted into the socio-economic context and the technology employed, since productivity is related to the method and effectiveness of the irrigation system (Silva & Azevedo, 2011). Another indicator that represents the social sphere or a response from the government, even minimal, when an extreme event is faced was determined using data from the Articulation of the Brazilian Semi-arid (ASA), that consists on the federal project to combat drought by the construction of water tanks (cisterns)-
\( n_c \) for the low-income population in risk situation. With this data, the averages for each region of the Northeast were determinate following the expression:

\[
AC = \frac{(n_c + n_l)}{1.000}
\]  

(5)

The methodology is based on the proposal Kienberger et al., (2009) where the concept of vulnerability is applied in order to diagnose most likely areas in a positive way or not, climate change, affecting various segments of a society.

3 Results and discussion

The parameter rain is quite variable in every region of the NEB, due to various weather systems of spatial-temporal scale, besides the topographical aspects. According to Oliveira et al. (2013), only the East Coast region has two rainy different quarters, the first occurs in the summer (December-February) and the second in the winter (June-August). In the research, according to the selected stations, the rainy season was between January-April, consequently the dry period was during July-October, as it can be observed in Figure 2.

Figure 2.

After determining the rainy period determination, the group was able to calculate the risk factor \( H \) using Equation 3. It is observed that the risk (Figure 3) vary from medium (0,76-1,37) to extreme (1,66 to 2,27), representing almost the whole semi-arid region of NEB. In addition, with the application of Eq. 3, the risk is extremely low. This result corroborates with Rao et al. (2015), that by analyzing precipitation climatology, they were able to determine values ranging from 300 to 600 mm.

Figure 3.

The risk calculated by the drought indicator (SPI) coupled with the drought decrees, highlights the potential injury caused by the extreme event during a certain time in the places considered high risk. This result is in accordance with Torres et al. (2012), that have determined a similar area with high values of socio-climatic vulnerability indicator (SCVI).

Also in Figure 3, there is the spatialization of Adaptive Capacity (AC), that the methodology was described in Eq. 6, considering that the used variables cover two levels suggested by the literature (Nelson et al. 2010), which would have a minimal action of the Government (adaptation) and technology involved to confront the dangerous event (resiliency). According to the IPCC AR5 report (2014), the difference between both is that the adaptation can be related to the preparation of the stressor event and the resilience is the way that the various areas of a society face the dangerous event. This is usually linked to the socio-economic characteristics and political actions employed to combat the stress factor (Hay & Mimura, 2006).

Also, it is observed that the entire region presents a average between medium and extreme AC, with values varying from 0,71-1,99; 1,99-5,42, respectively. Being justified by the amount of irrigated culture in the region, once that the productivity is
related to the method and effectiveness of the irrigation system (Silva & Azevedo, 2011). The lowest values are in the eastern
cost and West of NEB, comprising, basically, the state of Maranhão.

3.1 Application of the factorial analysis and construction of the agricultural sensitivity indicator (SeA)

As stated in sub-item 2.3, the factorial analysis (AF) seeks to reduce the amount of original variables on a smaller base of
latent data in such a way that this new base represents all variability of the original data. The adequacy analysis of the model
was given through the Kaiser-Meyer-Olkin test (KMO), as well as the Measurement of Sample Adequacy (MSA). The test
indicated that P1 presented a better KMO, 0.484 indicating a low ability of explanation between the factors and the variables.
On the other hand, the second simulation, in which the variables that presented MSA below 0.5 were removed, the KMO
improved its value, shifting to a value of 0.578. Therefore, the procedure for filling faults and removing variables that presented
inappropriate MSA contributed to the improvement of the factorial analysis result. Another evaluated test is the Bartlett’s Test
of Sphericity (BTS) that indicates the existence of a satisfactory relationship between the factors and the variables after
application of the analysis. It was considered a 5% level of significance for the test. In Table 2, all BTS values presented
statistical significance in both simulations. In this way, the construction of the factorial model was sample P1.

Table 2.

The commonality represents the variance proportion of a variable shared with the common factors in the factorial analysis.

Table 3 presents the values of P1 with MSA and commonality after the factors extraction. It is noticed that all variables present
values above 0.5 of MSA. Another important factor is the commonality. After the factors extraction, all of them increased.
The variables that presented commonality above 0.7, after the extraction of the factors, were the cotton tree, watermelon,
tomatoes and firewood with values of 0.774; 0.825; 0.638 and 0.653, respectively.

Table 3.

After knowing that the factors coefficients represent the correlation between the factor and the attribute (Table 4), it was
observed that the first factor is highly correlated with watermelon, whose value is 0.938. This research considered the loads
with the value of at least 0.6, despite the recommendation of 0.4 proposed by Figueiredo and Silva (2010). The second factor
is strongly related to tree cotton, while the third factor was firewood with a coefficient of 0.742. The variable that presents the
highest value for factor 4 was the tomato with 0.802 and, finally, the factor 5 was associated to the orange with a load of 0.667.
This AF presents a total variance of 42% for the five estimated relatively low factors compared to the values suggested by Hair
Jr et al. (2009). However, although the factorial model needs some adjust, it will be composed by five factors, that are also
called latent variables, and that represent segments of the agricultural production chain. In this way, the first factor represents fruit-exports, the main producer states are Bahia, Pernambuco and Rio Grande do Norte (EMBRAPA, 2006). The second factor is associated to the culture of dry farming, such as cotton that has specific hydro needs, where the appropriate irrigation system or rainfall in specific phenological periods can increase production (Silva et al., 2012).

Table 4.

The environmental impact or anthropogenic effect are highlighted in the third factor, where the largest coefficient is in the variable ‘firewood’ (0.742), responsible for the deforestation at the Caatinga region for raw material supply, mainly by ceramic and coal industries (Thomas et al., 2009). On the other hand, the fourth factor is associated to products composed by the food agribusiness, once that the tomatoes are the basis of many food industries (soup, sauce, juices, among others) having a significant increase in the consumer market, especially of fastfood. Approximately, 11% of the national tomato production is produced in the states of Bahia and Pernambuco, according to data provided by the Inter-Union Department of Statistics and Socio-economic Studies (DIEESE, 2010). Finally, the production of citrus, characterized the 5th factor, which is also an important component for food industry, being more than 90% of the northeastern production concentrated in the states of Bahia and Sergipe, according to EMPRAPA (2003).

After the construction of the factorial model, the SeA was calculated for each period. The result is shown in Figure 4, where low sensitivity values were observed in a few areas. The largest areas are located in the northwest of the region that comprises part of the states of Maranhão and Piauí.

Figure 4.

In the south of the region, the values ranged from moderate to extreme sensitivity, in areas covering the states of Bahia and part of southern Piauí. In the north part, there was a shrinkage in the values of SeA ranging from low to moderate (0.11 to 0.45). In the values in the east part ranged from moderate to high SeA. In Figure 4b, regarding the P2, where a change in the SeA pattern was observed, highlighting the variation on the northwest from low to high. The east of Bahia presented low SeA values, differing from figure 4a, where these areas presented values that ranged from moderate to high. This indicates that there was an increase in agricultural production in these areas, which leads to the belief that producers had technical guidance and technological investments.

3.2 Agricultural vulnerability to drought in northeast region of Brazil

Finally, in Figure 5, presented a spatialization of the Agricultural vulnerability indicator V. It is noticed that the vulnerability pattern suffered a change, where in P1, the area that presented a medium V (0.65-2.82) decreased and began to have a low vulnerability value (-0.97-0.74). The area of extreme vulnerability has virtually not changed, positioned almost in the center of the region, between the states of Bahia and Pernambuco.
In Figure 5a, it was observed that the most vulnerable areas are in the center-south of the NEB region, which comprises almost the entire state of Bahia, the states of Alagoas, Sergipe and part of Pernambuco, with high values of V. On the other hand, the northwest and north sectors of the NEB presented V ranging from low to moderate. It is also noticed that the V spatial pattern for the P2 period, mainly in the areas considered as high vulnerability, presented a low value (Figure 5b), in addition there was a reduction of the value in the areas considered high, where the range was between 2,82-17,81; became between 2,79-11,41.  

*Figure 5*

According to Buainain & Garcia (2015), over the years, the Brazilian government has invested in irrigated perimeters, increasing by 12,1% the planted area in the period from 2000-2011, as well as the temporary and permanent crops of 9,3 and 34,1%, respectively. It must be also highlighted in P2 the appearance of a high value area for V (2,79-11,43) in the Ceará state.

4. Conclusion

The results show that NEB has degrees of agricultural Vulnerability (V) between regular and high according to 2000 census, mainly in the southern region, comprising the state of Bahia. In addition, the risk of drought (H) is very high, even in the rainy period, especially in the central part of the NEB. Whereas, the AC factor showed that the semi-arid region of the northeast has a highly adaptive classification. Considering the variables used, the AC requires some improvement, by including a larger amount of sociodemographic variables. So, facing the concept of agricultural vulnerability to drought extremes adopted in this research, the main conclusions were:

1. There are areas where the risk of drought is extremely low, which is uncatalogued, in a strip of the east coast, covering the states from Bahia to Paraiba, and in the the far west compromising almost the whole the state of Maranhão.
2. Regarding the SeA, the P1 presented a greater statistical significance when applied the factorial model, which explained variance is low, 42%, considering the region's productive chain.
3. Concerning the adaptation capacity, the study shows that the NEB presented an adaptation between medium to extreme (0,24-5,42). Being the need to analyze broader socio-economic characteristics, such as the educational level of producer, emphasized.
4. In the scope of risk analysis, it was verified that a higher risk (0,76-2,26) area is located in the central strip of NEB.
5. The standard of vulnerability V, in addition to presenting improvement in P2, achieved, in general, good values, reinforcing that during the period there was improvement in the production, justified by the SeA of P2 and the AC adopted in the research.

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Samaniego, L., Kumar, R., Zink, M., Samaniego, L., Kumar, R. and Zink, M.: Implications of Parameter Uncertainty on Soil


Figure 1. Study area highlighting the political division of the region in micro-regions along with the precipitation stations of the National Water Agency (ANA)

Figure 2. Climate Characterization of rainfall in the Northeast of Brazil, between 1980 and 2011.
Figure 3. Spatialization of the drought Risk factor (H) for agriculture and Adaptation Capacity (AC) for events in northeastern Brazil.
Figure 4. Spatial distribution of agricultural sensitivity indicator to the Northeast of Brazil for the periods between 1990-1999 (a) and 2000-2010 (b).
Figure 5. Characterization of the agricultural vulnerability to drought at NEB for the periods of: 1990-1999 (a) and 2000-2010 (b).
Table 1. Source of the data used in the study

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Table 2. Kaiser-Meyer-Olkin test (KMO), Bartlett’s test of sphericity (BTS) and p-value.

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Table 3. Values referent to Measurement of Sample Adequacy (MSA) and commonalities, referring to P1.

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Table 4. Observed factors and their respective proportional and cumulative variances for agricultural production set

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| Variance proportional| 0,119  | 0,097   | 0,083   | 0,065   | 0,056   |
| Cumulative variance  | 0,119  | 0,216   | 0,299   | 0,364   | 0,420   |
Appendix A

Considering a set of $p$ variables, with $n$ observations for each variable, the values arrangement was obtained

$$ [x_i], i = 1, 2, ..., n, j = 1, 2, ..., p $$

(1)

From the following data group

<table>
<thead>
<tr>
<th>Individuals</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$\ldots$</th>
<th>$X_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_{11}$</td>
<td>$x_{12}$</td>
<td>$\ldots$</td>
<td>$x_{1p}$</td>
</tr>
<tr>
<td>2</td>
<td>$x_{21}$</td>
<td>$x_{22}$</td>
<td>$\ldots$</td>
<td>$x_{2p}$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$n$</td>
<td>$x_{n1}$</td>
<td>$x_{n2}$</td>
<td>$\ldots$</td>
<td>$x_{np}$</td>
</tr>
</tbody>
</table>

The factor analysis model assumes that each variable $X_j$ is linearly dependent of few random unobserved variables $F_1$, $F_2$, $\ldots$, $F_m$ ($m < p$), called common factors, and $p$ additional sources of variation $e_1$, $e_2$, $\ldots$, $e_p$, called errors or, sometimes, specific factors,

In particular, the model of factor analysis can be written as *

* Notice that in the present model, the average vector of each variable was disregarded simply to simplify the theoretical exposure, however without any loss of generality, it would be as if the group was working with the focused observations, which consists of subtracting from each observation the average value of the observations. The model, considering the average vector, would be:

$$ X_1 = a_{11}F_1 + a_{12}F_2 + \ldots + a_{1m}F_m + e_1 $$

$$ X_2 = a_{21}F_1 + a_{22}F_2 + \ldots + a_{2m}F_m + e_2 $$

$$ \vdots $$

$$ X_p = a_{p1}F_1 + a_{p2}F_2 + \ldots + a_{pm}F_m + e_p $$

Otherwise,

$$ X_j = a_{j1}F_1 + a_{j2}F_2 + \ldots + a_{jm}F_m + e_j $$

Where $X_j$ is the $j^{th}$ variable, $a_{j1}$, $a_{j2}$, $\ldots$, $a_{jm}$ are the loads of factors for the $J^{th}$ variable and $F_1$, $F_2$, $\ldots$, $F_m$ are $m$ common non-correlated factors, where $m$ is smaller than $P$. 


The observed p values $X_p$ are expressed in terms of $p + m$ non-observable random variables $(F_1, F_2, ..., F_m; \varepsilon_1, \varepsilon_2, ..., \varepsilon_p)$. This distinguishes the factorial model from the multiple regression model, in which independent variables can be observed, and whose positions are occupied by $F$ in the factorial model.

So in a matrix, it would be

$$X_{(px1)} = \Lambda_{(pxn)} \ast F_{(mx1)} + \varepsilon_{(px1)}$$

(3)

A direct verification of the factorial model, from the observations $X_1, X_2, ..., X_p$, is prevented by so many unobservable quantities. However, with some assumptions imposed on random vectors, $F$ and $\varepsilon$, the factorial model implies certain covariance relationships, which can be verified. Therefore, the $F$ and $\varepsilon$ vectors must meet the following conditions:

$$E(F) = 0_{(mx1)}$$
$$Cov(F) = E(FF^\prime) = I_{(mxm)}$$
$$E(\varepsilon) = 0_{(px1)}$$
$$Cov(\varepsilon) = E[\varepsilon\varepsilon^\prime] = \psi_{(pxp)}$$

where $\psi$ is a diagonal array and $F$ and $\varepsilon$ are independent, So,

$$Cov(\varepsilon, F) = E(\varepsilon F^\prime) = 0_{(pxm)}$$

(4)

These assumptions and the relationship in (3) constitute the so-called orthogonal factor model.

If the authors admit that the $F$ factors are correlated, being Cov(f) not diagonal, the result be the so-called oblique factors model, This model will not be discussed in this work.

Based on the previous assumptions, it is possible obtain the structure of the $X$ Covariances matrix, which will be represented by $\Sigma$,

There is $XX^\prime = (\Lambda F + \varepsilon) (\Lambda F + \varepsilon)^\prime$

$$= (\Lambda F + \varepsilon) [(\Lambda F)^\prime + \varepsilon^\prime]$$
$$= \Lambda F (\Lambda F)^\prime + \varepsilon (\Lambda F)^\prime + \Lambda F \varepsilon^\prime + \varepsilon \varepsilon^\prime$$

So, according to (4):

$$\Sigma = Cov(X) = E(XX^\prime)$$
$$= \Lambda E(FF^\prime) \Lambda^\prime + E(\varepsilon F^\prime) \Lambda^\prime + \Lambda E(Fe^\prime) + E(\varepsilon \varepsilon^\prime)$$
$$= \Lambda I \Lambda^\prime + 0 + 0 + \Psi$$
$$= \Lambda \Lambda^\prime + \Psi$$

(5)

An easier way to understand, that only uses variance properties, is capable to reach the same result from the relationship (2):

$$X_j = a_{j1}F_1 + a_{j2}F_2 + \cdots + a_{jm}F_m + e_{jm}$$

By applying the properties of the variance, and based on the assumptions (eq. 2,3):

$$V(X_j) = a_{j1}^2 V(F_1) + a_{j2}^2 V(F_2) + \cdots + a_{jm}^2 V(F_m) + V(e_j) = a_{j1}^2 + a_{j2}^2 + a_{jm}^2 + V(e_j)$$
where, \( a_{j1}^2 + a_{j2}^2 + a_{jm}^2 \), i.e., \( \Lambda \Lambda' \), is called the commonality of the variable \( X_j \) (the part of its variance that is related to the common factors) whereas \( V(e_j) \) is called the specificity of \( X_j \) (the part of its variance that is not related to the common factors).

So, the covariance structure will be:

5

a. \( \text{Cov}(X) = \Lambda \Lambda' + \Psi \) ou

\[
\text{Var}(X_i) = a_{i1}^2 + a_{i2}^2 + \cdots + a_{im}^2 + \Psi_i
\]

\[
\text{Cov}(X_i, X_k) = a_{i1}a_{k1} + \cdots + a_{im}a_{km}
\]

b. \( \text{Cov}(X, F) = \Lambda \)

\[
\text{Cov}(X_i, X_k) = a_{ij}
\]

10

It can also be established that the correlation between \( X_j \) and \( X_j' \) is

\[
\rho_{jj'} = a_{j1}a_{j'1} + a_{j2}a_{j'2} + \cdots + a_{jm}a_{j'm}
\]

Consequently, two variables will only be highly correlated if they have high loads on the same factor,