REPLY TO REFEREES AND GUIDE TO THE REVISION OF THE PAPER

Natural Hazards and Earth System Sciences

Title: Development of Bridge Failure Model and Fragility Curves for Infrastructure Overturning and Deck Sliding due to Lahars

Authors: Joaquín Dagá, Alondra Chamorro, Hernán de Solminihac, Tomás Echaveguren

MS N°: nhess-2017-330

Anonymous Referee #1

Point 1a: The improper use of pronouns and adjectives causes difficulty in following the logic of the discussions. I recommend being managed this matter prior to discussing the details of the manuscript. Examples of the improper pronouns and adjectives are follow Page 3, line 30, 'latter’ what this means? fragility curve, fragility probability or others? Page 3, line 21, 'they’ what this means? Page 4, line 5, 'they’ what this means? Page 18, line 6, 'they'. I think it is better to be replaced to ‘we’.

We appreciate the comments of Referee #1 and we realize that several pronouns and adjectives were not used properly. The paper was edited and grammar was improved.

On page 3, line 21, the use of the term ‘they’ refers to ‘debris flows’. The text was corrected as follows:

“Debris flows are capable of transporting gravel-sized debris in suspension, and their concentration of solid particles ranges between 75 and 80 % in weight or 55 and 60 % in volume.”

On page 3, line 30, the term ‘latter’ refers to fragility curves; therefore, the word ‘latter’ was replaced by ‘fragility curves’. The corrected text reads as follows:

“In order to incorporate the uncertainty of the characteristics of lahar flows and the bridge engineering design (X), the use of fragility curves to quantify the probability of bridge failure due to lahars is proposed. Fragility curves express the probability that a system exceeds different damage states (ds_i) as a function of the hazard intensity (IM) (See Eq. 1).”

On page 4, line 5, the word ‘they’ refers to ‘fragility curves’. The text was adjusted as follows:

“Fragility curves can also be developed using an analytical approach through models that characterize the limit state of the element, based on probabilistic and deterministic variables defining the system.”

Section ‘Acknowledgements’ (Page 18, line 6) was rewritten in first person. The corrected text reads as follows:

“Likewise, we express our gratitude to the institutions that participated and contributed to this research project, especially to: […]”

Point 1b: Other minor suggestions. Page 1, line 21, insert ’simulation’ following to ’Monte Carlo’.
The term ‘simulations’ was included following to ‘Monte Carlo’ as suggested. The new text reads as follows:

“Monte Carlo simulations were applied to quantify the probability of bridge failure given by different lahar depths.”

**Point 1c: Page 4, line 13, I cannot understand why ’on the other hand’ used here (both Tsubaki et al. and Wilson et al. use the flow depth as the hazard intensity measure).**

We agree with Referee #1, the term ‘on the other hand’ generates confusion since both Tsubaki et al. and Wilson et al. recommend using the flow depth as the intensity measure. The term ‘on the other hand’ was removed from the sentence. The new text reads as follows:

“Tsubaki et al. (2016) use the same variable (flow depth) for measuring the flood intensity when developing embankment fragility curves. Wilson et al. (2014) propose the flow depth as one of the potential intensity measures for developing fragility curves related to lahar flows as well.”

**Point 1d: Page 4, line 13-15,’ Moreover, the existing velocity and scour models use the flow height’ doesn’t make sense.**

Again we agree with Referee #1 and the sentence was not written properly. The purpose of the phrase was to provide a justification for the use of the flow height as a measure of lahar intensity. The original sentence was modified as follows:

“In this paper the lahar depth was proposed as lahar hazard intensity, considering that this variable is correlated to other lahar flow characteristics, such as velocity and scour demand (Arneson et al., 2012).”

**Point 1e: Pointed-out above are all minor points, but make difficult to follow the logic flow of the manuscript so discussing the detail of the present manuscript may cause many inessential discussion caused by misinterpretation of the authors’ intentions. Thus, I suggest to revises the English language first.**

As Referee #1 pointed-out, we agree that grammar was misleading in the initial version of the article. Language was revised and edited throughout the entire text to avoid misinterpretation.

Once again, the authors appreciate the comments made by Referee #1 and believe that the manuscript improved significantly after including the suggested adjustments.
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**Anonymous Referee #2**

**Point 2a:** As the other reviewer states, the grammar makes it difficult to follow the logic of this manuscript. English proof-reading is needed to ensure the minor issues of tense, pronouns and adjectives are addressed and do not confuse the reader. The use of ‘on the other hand’ (Page 3, line 9; Page 4, line 5; Page 8, line 2; Page 10, line 5; Page 16, line 11 and more) also causes a lot of confusion.

We completely agree with Referee #2 and we realize that certain pronouns and adjectives cause confusion to the reader. We made an effort to edit and significantly improve grammar. For this, we removed from the text terms like ‘on the other hand’ and ‘the latter’. The following table synthesizes some of the improvements included in the text:

<table>
<thead>
<tr>
<th>Page and line</th>
<th>Original Manuscript</th>
<th>New Manuscript</th>
</tr>
</thead>
<tbody>
<tr>
<td>Page 3, line 9</td>
<td>“On one hand, there are debris flows, highly viscous slurries of sediment and water. They are capable of transporting gravel-sized in suspension, and their concentration of solid particles ranges between 75 and 80 % in weight or 55 and 60 % in volume. On the other hand, there are hyper-concentrated flows, which have high suspended fine contents, predominantly due to fluid motion and properties. Their solid concentrations reaches 55 to 60 % in weight and 35 to 40 % in volume (Pierson et al., 2009).”</td>
<td>“Debris flows are highly viscous slurries of sediment and water. Debris flows are capable of transporting gravel-sized debris in suspension, and their concentration of solid particles ranges between 75 and 80 % in weight or 55 and 60 % in volume. Hyper-concentrated flows have high-suspended fine contents, predominantly due to fluid motion and properties. The solid concentrations of hyper-concentrated flows can represent up to 55 to 60% of the total weight, and 35 to 40% of the total volume (Pierson et al., 2009).”</td>
</tr>
<tr>
<td>Page 4, line 5</td>
<td>“On the other hand, curves can be based on experts’ opinion.”</td>
<td>“Fragility curves can be based on experts’ opinions as well.”</td>
</tr>
<tr>
<td>Page 8, line 2</td>
<td>“On the other hand, the debris transported by the flows are accumulated in the bridge piers, thus creating an additional obstruction to the flow.”</td>
<td>“Debris transported by the flows accumulates in the bridge piers, creating an additional obstruction to the flow.”</td>
</tr>
<tr>
<td>Page 10, line 5</td>
<td>“On the other hand, the numerical solution methods include the Monte Carlo simulation (MCS) and the response surface method (RSM).”</td>
<td>“Numerical solution methods include the Monte Carlo simulation (MCS) and the response surface method (RSM).”</td>
</tr>
<tr>
<td>Page 16, line 11</td>
<td>“On the other hand, the analysis of the models and equations used in the limit state...”</td>
<td>“The analysis of the models and equations used in the limit state functions demonstrates...”</td>
</tr>
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</table>
functions allow concluding that the lahar depth is the main variable in the quantification of lahar loads and bridge capacity to these flows.”

that the lahar depth is the main variable in the quantification of lahar loads and bridge capacity to response to these flows.”

Point 2b: Figures 3 - 5 need to be modified (thinner line weights, different symbols, patterned lines) to ensure the graphics are easily readable in greyscale. Figure 1 is well drawn and designed, although definition of Q; qmin; qmax is needed.

We appreciate and agree with Referee #2 that graphs needed improvement to be easily readable in greyscale. Graphs were improved considering thinner lines, patterned lines and different symbols, as suggested.

With regard to the terms Q, qmin and qmax, the authors realized that these were not required in the graphs so they were therefore eliminated.

Adjusted figures are presented below:

**Figure 1:** Free-body diagram of bridge resisting and demanding forces and moments in the presence of a lahar.
Figure 3: Fragility curves for bridge infrastructure overturning and deck sliding due to lahars.

Figure 4: Fragility curves for one-span bridges (C1) and multiple-spans bridges (C2) due to lahars.

Figure 5: Analytical and parameterized fragility curves for one-span bridges (C1) and multiple-spans bridges (C2) due to lahars.

Point 2c: Page 2, line 5: "This implies less exposure and therefore, vulnerability...". This is wrong, risk is generally considered as a function of the hazard, exposure and vulnerability. In this example, the exposure and vulnerability are the same but the hazard is lower - resulting in lower risk. One could also argue that exposure is lowered, but this will not lower the vulnerability.

We sincerely appreciate the comment from Referee #2 and we realize that the sentence lead to confusion. As stated by Referee #2, risk is generally considered to be a function of the hazard, exposure and vulnerability. In the original text we considered vulnerability as a function of exposure, which is also agreed by some authors (Wilson et al., 2014). The text was adjusted as suggested by the Referee #2 given that most literature agrees with the fact that exposure and vulnerability are not necessarily correlated. For this the UNISDR (2009)
definition of risk was incorporated as a reference, where risk is considered as a function of the hazard, exposure and vulnerability. The improved text reads as follows:

“Lava and pyroclastic flows destroy the infrastructure but, in contrast, their probability of occurrence is low and their influence area is small (Wilson et al., 2014). This implies a lower hazard intensity and exposure and, therefore, a lower risk of lava and pyroclastic flows on the infrastructure, considering that risk is a function of the hazard, exposure and vulnerability (UNISDR, 2009).”

New references:


Point 2d: Page 2, paragraph 3: "From available literature..." not much literature has been explicitly surveyed here - only examples of risk management software. The Wilson (2014) review is quite extensive, but the manuscript would benefit from a broader review of available literature on bridge fragility functions.

We agree that limited bridge fragility models were referred to in the text, although others than Wilson et al. (2014) were reviewed by the authors initially. In order to present a broader perspective about the effects of volcanic hazard on different infrastructures as well as existing bridge fragility functions due to other hazards, authors refer to several examples of these models. The paragraph was improved as follows:

“Several authors have calibrated fragility curves for buildings and electrical transmission systems, considering the vulnerability of both to volcanic hazard (Spence et al., 2005; Spence et al., 2007; Jenkins and Spence, 2009; Zuccaro and De Gregorio, 2013). Wilson et al. (2017) developed road infrastructure fragility curves due to tephra fall, without analyzing the effect of lahars on bridges. Fragility curves are commonly integrated in available risk modelling tools. For example, in the United States, the Federal Emergency Management Agency (FEMA) [...]”

New references:


Point 2e: Page 5, line 1: The foundation has no piles. Is this justified by bridge designs (especially in your study area)? It may be a valid assumption, but the authors need to
justify this with data (i.e. in the bridges used in subsequent sections, did any have piles?).

We completely agree with Referee #2 that this assumption, although valid, must be justified. Indeed, in this paper bridge design criteria of Chile are used. The proposed failure model can be adapted and calibrated for different bridge design standards. To justify the assumption that the modeled bridge does not have piles, the Chilean bridges exposed to the volcanic hazard were analyzed; we demonstrated that 88% of the Chilean bridges exposed to the volcanic hazard from the Villarrica and Calbuco volcanoes do not have piles. The new text reads as follows:

“The proposed failure model can be adapted to different bridge design criteria. In this paper, the Chilean design standards are considered for the fragility curves calibration. Thus, the proposed model assumes that the foundation has no piles. This assumption is based on the fact that 88% of the bridges exposed to the volcanic hazard from the Villarrica and Calbuco volcanoes do not have piles (Moreno, 1999; Moreno, 2000). Additionally, it assumes a simple support of the superstructure on the piers and abutments.”

New references:

Moreno, H.: Mapa de peligros del volcán Calbuco, Región de Los Lagos, Servicio Nacional de Geología y Minería, Documento de Trabajo Nº12, map scale 1:75.000, 1999.


Point 2f: Page 6, line 15: So you are not explicitly modelling the effect of scour on the resisting moment? Destabilisation from erosion (mentioned on Page 7, lines 1-7) would surely have a large role on changing the location or size of the moment. How is this accounted for?

We appreciate the comment of Referee #2 and we realize that this point was not well explained in the original manuscript. The capacity of the bridge to resist lahar loads depends on the design and condition of the bridge. The supply function (resistant moment) of the proposed failure model considers only the design criteria, without considering the bed condition. However, the scour generated by the lahar is considered in the demand function. The scour demanded by the lahar produces a greater hydrodynamic force and overturning moment. Therefore, the effect of scour causes an increase in the probability of bridge failure. This is explained in two parts of the new manuscript:

“The scour produced by lahar flows near the foundations contributes to a greater vulnerability of these bridge components, since the lahars produce destabilization and weakening around the foundation of piers and abutments. If there is scour in the bed, the foundation of the pier or abutment will be exposed to a higher hydrodynamic pressure. This load is higher in the case of lahars, given their greater density and velocity in relation to normal floods. A greater scour demand will imply a larger surface affected by the hydrodynamic pressure. In turn, this means a greater resulting hydrodynamic force ($F_{wl}$) and, therefore, a greater moment associated with this force ($M_{wl}$).
The infrastructure capacity to oppose overturning depends on the bridge elements’ design and condition, including the bridge geometry, materials and the scour’s design ($Y_{c-o}$ and $Y_{e-o}$). Thus, the lahar loads on the bridge and the scour are considered only in the demand function (overturning moment $M_v$). The resistant moment ($M_r$) of the infrastructure to lahars is given by the weight ($W$) of the pier or abutment and the elements that are supported on it. [...]”

**Point 2g:** Page 7, line 25: The estimation of velocity Mannings formula is based on the assumptions of a one-dimensional, steady state flow, which is unlikely around bridges. Also, how was the effect of rheology on the flow accounted for? The velocity (and height) will depend on the rheology of the flow; this should probably be accounted for in the Monte-Carlo simulations.

We agree with Referee #2 that the Manning formula considers certain assumptions that limit its applicability for modeling lahar velocity around bridges. In order to improve the estimation of the lahar mean velocity, the Chen formula was used instead of the Manning formula. The Chen formula is recommended for fully dynamic debris flows because it incorporates the rheology of the lahar through the consistency index ($\mu_{Lahar}$). To explain this, the following paragraph was added in the paper:

“First, the lahar mean velocity ($v_{Lahar}$) is quantified with the Eq. (6), suggested by Chen (1983; 1985) for a fully dynamic debris flow in a channel with an arbitrary geometric shape. For this case, a rectangular flow is assumed. This formula incorporates the rheology of the lahar through the consistency index ($\mu_{Lahar}$), which was quantified by Laenen and Hansen (1988) for the case of lahars.

$$v_{Lahar} = \frac{2}{5} \left( \frac{Y_{Lahar}}{\mu_{Lahar}} \right)^{1/2} \left( \frac{A_{Lahar}}{P_{Lahar}} \right)^{3/2} \frac{1}{\gamma_{Lahar}}$$

(6)”

The value of the consistency index ($\mu_{Lahar}$) is also shown in Table 1. The use of the Chen formula for the estimation of lahar velocity generated minor changes in the bridge fragility curves. These changes were incorporated into the new manuscript.

New references:


**Point 2h:** Page 7, line 27-29: How valid is a ’clear fluid’ scour model for lahars? Is this model used? The grammar is unclear on page 8 (On the other hand), but if it isn’t used - why is it mentioned in such detail?

We appreciate the comment of Referee #2 and we understand his/her question regarding the applicability of the mentioned scour models for the case of lahars. Originally, the scour
models proposed by HEC-18 were valid for ‘clear fluids.’ The NCHRP adjusted the scour model proposed by the HEC-18 empirically to estimate the scour generated by debris flows and lahars. To explain this, the following paragraph was added to the paper:

“Debris transported by the flows accumulates in the bridge piers, creating an additional obstruction to the flow. To incorporate the debris accumulation, the scour demand on the piers ($Y_{c-d}$) is modelled with Eq. (8) and (9) of the NCHRP (2010). The equations proposed by the NCHRP adjust the scour model proposed by the HEC-18 to estimate the scour generated by debris flows and lahars.”

**Point 2i: Page 8, how is the bending moment calculated? Where is the impact force located?** Debris tend to collect on the surface of the flow, increasing the moment - the magnitude of this effect may be important (particularly for deck sliding).

We agree with Referee #2 that lahar demand and bridge supply models are not explained in detail, thus causing confusion. In order to clearly describe the equations used in the supply and demand functions, four subsections were incorporated into section ‘4.1 Physical models to estimate limit state functions.’ In the first subsection, velocity and hydrodynamic pressure models for lahar are detailed. In the second subsection, scour models are presented. The third subsection explains the methodology to quantify the demand function; this includes the infrastructure overturning moment and the deck tangential force. In the fourth subsection, supply functions are explained in detail.

Regarding the debris impact height ($h_{imp}$), we made a mistake in describing the probability distribution of this variable. In the Monte Carlo simulations we considered a triangular distribution with mode equal to the lahar height ($h_{Lahar}$) for the impact height instead of a uniform one.

In the following paragraphs of the subsection ‘4.1.3 Infrastructure overturning moment and deck tangential force,’ the methodology for quantifying the overturning moment and debris impact height is explained:

“The overturning moment ($M_v$) produced by lahars on the bridge infrastructure is given by the sum of the hydrodynamic moment ($M_{wi}$) and the debris impact moment ($M_i$). The tangential force ($F_t$) on the deck corresponds to the sum of the resulting force from the hydrodynamic pressure on the deck ($F_w$) and the debris impact force ($F_i$). Considering the pressure model showed in Eq. (7), the hydrodynamic moment generated by the lahar on the infrastructure ($M_{wi}$) can be estimated. In the case of infrastructure, the hydrodynamic moment is separated into two parts: the foundation and the column. This separation is supported by the fact that these elements have different geometry and that the pressure has a triangular distribution over the foundation and trapezoidal distribution over the column (Fig. 1).

\[ M_{wi} = M_{w,found} + M_{w, column} = F_{w,found}Y_{w, found} + F_{w, column}Y_{w, column} \]  

(12)

The resulting hydrodynamic force exerted by the lahar on the foundation ($F_{w,found}$) and the height at which this force acts with respect to the turning axis ($Y_{w,found}$) are given by Eq. (13) and Eq. (14):

\[ F_{w,found} = LC_D \left( \frac{Y_{Lahar}}{2g} \right) v_{Lahar}^2 \left( \frac{Y_{sd}^2}{h_{Lahar} + Y_{sd}} \right), \]  

(13)
\[ y_{w, fuond} = Y_{so} - \frac{Y_{sd}}{3}, \quad (14) \]

The hydrodynamic force on the column \((F_{w, column})\) and its application point \((y_{w, column})\) depend on if the height of the lahar exceeds the height of the column or not. To incorporate this, the variable \(h^*\) was defined, which is given by the minimum between the lahar height \(h_{Lahar}\) and the column height \(h_{Design}\).

\[ F_{w, column} = bC_D \left( \frac{Y_{Lahar}}{2g} \right) v_{Lahar}^2 \left( \frac{h^* + 2h^*Y_{sd}}{h_{Lahar} + Y_{sd}} \right), \quad (15) \]
\[ y_{w, column} = Y_{so} + \frac{\left( h^* + \frac{h^*}{2} \right)^2 + \frac{3}{2}}{\left( Y_{sd} + \frac{h^*}{2} \right)}, \quad (16) \]

[...]

The moment of debris impact \((M_i)\) on the infrastructure with respect to the rotation axis is shown in Eq. \((19)\). This indicates that if the impact height \((h_{imp})\) is greater than the infrastructure \((h_{Design})\), the associated moment is zero. For the impact height, a triangular distribution with the mode equal to the lahar height is assumed, considering that the debris tends to collect in the flow surface (Zevenbergen et al., 2007).

\[ M_i = \begin{cases} 
    v_{Lahar} \sqrt{Y_{Grava} \frac{4}{3} \pi \left( \frac{D_{Grava}}{2} \right)^3} (h_{imp} + Y_{so}) & \text{if } h_{imp} \leq h_{Design} \\
    0 & \text{if } h_{imp} > h_{Design} 
\end{cases}, \quad (19)" \]

Point 2j: In Table 1, the variables of GammaGravel; DGravel; himp; eSuper are not mentioned in the manuscript. How are they used in the Monte-Carlo simulations?

We appreciate the comment of Referee #2 and we realize that these variables were not well explained in the original manuscript. By improving the explanation of the supply and demand functions in section ‘4.1 Physical models to estimate limit state functions’, the explanation of all variables was also improved considerably.

The gravel specific weight \((Y_{Gravel})\), the gravel diameter \((D_{Gravel})\) and the impact height \((h_{imp})\) are used to estimate the force and moment of debris impact \((F_{1s} \text{ and } M_i)\). The superstructure thickness \((e_{Super})\) is used to estimate the force exerted by the superstructure on each foundation \((W_{Super})\) and the friction between the superstructure and the infrastructure \((F_r)\). In order to explain this, the following paragraphs and equations were added in section ‘4.1 Physical models to estimate limit state functions’:

“In order to quantify the hydrodynamic force of the lahar on the deck \((F_{ws})\), three cases should be considered: (1) the lahar height is lower than the bridge clearance, (2) the lahar height is greater than the clearance but lower than the roadway level, (3) the lahar height is greater than the roadway level. In the model, the roadway level is given by the sum of the infrastructure height \((h_{Design})\), and the superstructure thickness \((e_{Super})\).

\[ F_{ws} = \begin{cases} 
    0 & \text{if } h_{Lahar} < h_{Design} \\
    L_{Bridge} bC_D \left( \frac{Y_{Lahar}}{2g} \right) v_{Lahar}^2 \left( \frac{h_{Design} - h_{Lahar}^2}{h_{Lahar} + Y_{sd}} \right) & \text{if } h_{Design} \leq h_{Lahar} < h_{Design} + e_{Super} \\
    L_{Bridge} bC_D \left( \frac{Y_{Lahar}}{2g} \right) v_{Lahar}^2 \left( \frac{2h_{Design}e_{Super} + e_{Super}^2}{h_{Lahar} + Y_{sd}} \right) & \text{if } h_{Lahar} \geq h_{Design} + e_{Super} 
\end{cases}, \quad (17) \]
To quantify the impact of debris on the bridge, the model of Haehnel and Daly (2004) is used. This model assesses the impact force through a one-degree-of-freedom system assuming a rigid structure. Thus, the impact force of gravel transported by a lahar on the bridge is based on the flow velocity ($v_{\text{Lahar}}$), the specific weight of the gravel ($\gamma_{\text{Gravel}}$), the gravel diameter ($D_{\text{Gravel}}$) and the contact stiffness of collision ($\hat{k}$). Debris impact force on the deck ($F_{\text{Is}}$) is given by Eq. (18).

\[
F_{\text{Is}} = \begin{cases} 
0 & \text{if } h_{\text{imp}} < h_{\text{Design}} \\
 v_{\text{Lahar}} \sqrt{\hat{k} \gamma_{\text{Gravel}} \frac{4}{3} \pi \left( \frac{D_{\text{Gravel}}}{2} \right)^3} & \text{if } h_{\text{Design}} \leq h_{\text{imp}} < h_{\text{Design}} + e_{\text{Super}}, \\
0 & \text{if } h_{\text{imp}} \geq h_{\text{Design}} + e_{\text{Super}}, 
\end{cases} \quad (18)
\]

The moment of debris impact ($M_{\text{i}}$) on the infrastructure with respect to the rotation axis is shown in Eq. (19). This indicates that if the impact height ($h_{\text{imp}}$) is greater than the infrastructure ($h_{\text{Design}}$), the associated moment is zero. For the impact height, a triangular distribution with the mode equal to the lahar height is assumed, considering that the debris tends to collect in the flow surface (Zevenbergen et al., 2007).

\[
M_{\text{i}} = \begin{cases} 
v_{\text{Lahar}} \sqrt{\gamma_{\text{Grava}} \frac{4}{3} \pi \left( \frac{D_{\text{Grava}}}{2} \right)^3} (h_{\text{imp}} + Y_{\text{so}}) & \text{if } h_{\text{imp}} \leq h_{\text{Design}}, \\
0 & \text{if } h_{\text{imp}} > h_{\text{Design}}, 
\end{cases} \quad (19)
\]

The model considers that the weight of the superstructure is distributed uniformly in all its supports (NA). Thus, the force exerted by the superstructure on each foundation is:

\[
W_{\text{Super}} = \frac{(\gamma_{\text{Super}})(L)(L_{\text{Bridge}})(e_{\text{Super}})}{\text{NA}}, \quad (22)
\]

Finally, the force that opposes the deck sliding corresponds to the friction between the superstructure and the infrastructure. This force is given by the Eq. (24):

\[
F_{r} = \mu_s N_{\text{Super}} = \mu_{\text{Super}} (\gamma_{\text{Super}})(L)(L_{\text{Bridge}})(e_{\text{Super}}), \quad (24)
\]

**Point 2k:** In equations 19 and 20, the important parameters $n_a$; $n_e$; $x_a$; $x_e$ are not defined or fully explained. Although $n_e$ might be assumed to be 14, what is the value of $n_a$?

We agree with Referee #2 that the validation parameters should be defined and explained in the manuscript. In order to improve validation description, the following paragraph was added:

“Where $n_a$ is the number of bridges evaluated analytically with a lahar with intensity $h_{\text{Lahar}}$ (10,000 simulations), $x_a$ the number of simulations in which the bridge fails considering an intensity $h_{\text{Lahar}}$ in the analytical model; where $n_e$ is the number of bridges that were reached empirically by lahars with intensity $h_{\text{Lahar}}$ and, $x_e$ the number of bridges that were destroyed empirically by lahars with intensity $h_{\text{Lahar}}$. The data and results of the test statistic $Z_{\text{test}}$ obtained for each hypothesis test associated with each point are shown in Table 3 and Table 4.”

Additionally, Tables 2 and 3 of the original manuscript were modified to clearly show the
values of the empirical validation parameters ($n_a, n_e, p_a, p_e$) of the fragility curves.

**Point 2l:** On page 15, line 15 and on: At these p/Z-values, the null hypothesis is not rejected. However, this does not establish that empirical and analytical proportions are the same due to the low sample size. The significance has not been fully tested, as you have not established the statistical power of the samples.

We really appreciate the comment of Referee #2 and we realize that the validation should be improved. In the new paper, the section ‘6 Validation of bridge failure model and fragility curves due to lahars and analysis of results section’ was separated into three subsections. The first subsection corresponds to the validation of the bridge failure model. In this first subsection, the limit state functions defined for the system are empirically validated. For this, data from historical lahars of Chile is used. Considering the attributes of the historical lahars and bridges reached, the model quantifies the net moment and net force exerted by the flow on the bridge. If the demand moment or force exceeds that of supply, the model indicates that the analyzed bridge failed due to that historical lahar. The model result for each bridge (failure/not failure) is compared with that indicated in the damage reports. The 15 historical cases evaluated analytically with the failure model, considering the specific inputs of the system, have the same state of damage (failure/no failure) as that reported experimentally by the agencies.

In the second subsection, the empirical validation of the fragility curves is presented. Through an investigation of reports of historical lahars and their damages in bridges, six empirical points of probability of failure were built ($h_{Lahar}, p_e$). These points were built from 15 reports of bridges reached by lahars in Chile. In the new manuscript, a level of significance of 5% was defined for the statistical test that compares the probability of analytical failure and the probability of empirical failure. Considering the defined significance level, Z-test values of all the empirical points evaluated are within the acceptance region. We therefore concluded that it is possible to accept the null hypothesis $H_0$, which establishes that empirical bridge failure probability due to lahars is equal to that indicated by the analytical model, with a 5 % significance level. Additionally, an effort was made to improve the explanation of the process of statistical validation of the fragility curves.

The last paragraph of the fragility curves validation reads as follows:

"Once the test statistic $Z_{test}$ of every hypothesis test associated with each point is calculated, it is compared with a significance level $\alpha$ for validation. For the fragility curve validation, a significance level of 5% was considered. The critical value ($Z_{critical}$) of $\pm1.96$ delimits the region of acceptance and rejection of the null hypothesis. If the test statistic $Z_{test}$ is located in the acceptance region [-1.96; +1.96], the null hypothesis $H_0$, stating that the bridge empirical failure probability due to lahars is equal to that obtained by the parameterization ($H_0; p_a = p_e$); this should be accepted with that significance level. In this case, the $Z_{test}$ values of all the empirical points evaluated are within the acceptance region. The maximum absolute value obtained from $Z_{test}$ was 0.44, for one-span bridges reached by lahars of 2.50 m. Therefore, we conclude that it is possible to accept the null hypothesis $H_0$, which establishes that empirical bridge failure probability due to lahars is equal to that indicated by the analytical model, with a 5 % significance level."

In the third subsection, the results obtained in the validation process of the failure model
and the fragility curves are analyzed.
Once again, the authors appreciate the comments made by Referee #2 and believe his/her suggestions and observations have greatly improved the manuscript.