Probability assessment on the recurring Meishan earthquake in central Taiwan with a new non-stationary analysis

J. P. Wang and X. Yun

Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Kowloon, Hong Kong

Received: 31 March 2014 – Accepted: 18 May 2014 – Published: 31 July 2014

Correspondence to: J. P. Wang (jpwang@ust.hk)

Published by Copernicus Publications on behalf of the European Geosciences Union.
Abstract

From theory to experience, earthquake probability should be increasing with time as far as the same fault is concerned, rather than being a stationary process or independent of the date of the last occurrence. With a new non-stationary model, we evaluated the earthquake probability associated with the Meishan fault in central Taiwan, a growing concern to the local community given a relatively short return period reported (i.e., around 160 years). The analysis shows that on the condition that the earthquake has not recurred by the end of year 2014, the earthquake probability in the next 50 years could be around 0.3 (mean value), with a 95% confidence interval from 0.26 to 0.36.

1 Introduction

In 1906, an $M_L = 7.1$ earthquake, later referred to as the Meishan earthquake, struck central Taiwan and caused severe damage around the region. Recently, the return period of such events was reported at 162 years (Wang et al., 2012), and the relatively short return period makes the Meishan fault a growing concern to the local community. Under the circumstance, a few studies focusing on earthquake potential and seismic hazard associated with the recurring Meishan earthquake were reported (Wu et al., 2009; Wang et al., 2012, 2013), with the same objective to mitigate earthquake risk around the region close to the active fault.

The Poisson process is commonly used for assessing earthquake probability (e.g., Weichert, 1980; Ang and Tang, 2007; Ashtari Jafari, 2010). By definition, the approach is a “memory-less” model (Devore, 2008), meaning that the probability is a function of the length of time only, but being independent of the date of the last occurrence. However, such a “memory-less” process seems not reflecting the reality very well because earthquake occurrence should be somehow related to the date of the last occurrence. For example, it should be very unlikely for the 2011 Tohoku earthquake in Japan to recur in the next decade, but more likely in next hundreds of years.
Given the “shortcoming” of the stationary Poisson model, a few non-stationary models were proposed for earthquake probability assessment, such as renewal models and Markov models (Hagiwara, 1974; Veneziano and Cornell, 1974; Nishioka and Shah, 1980; Savy et al., 1980; Kiremidjian and Anagnos, 1984). But similar to the Poisson model, somehow those non-stationary models were also developed from an empirical perspective, but not on the basis of earthquake mechanisms like the elastic rebound theory (Reid, 1910; Keller, 1996).

Given the Meishan fault in central Taiwan a growing concern to the local community, this paper is aimed at evaluating the earthquake probabilities with a new non-stationary analysis. Different from others, the key to the new model is to analyze the stress on the fault plane at a given time, then comparing it to the failure stress to evaluate the earthquake probability at that time. In addition to the methodology, this study applied the non-stationary approach to the Meishan fault in central Taiwan, estimating that the probability is around 0.3 for the Meishan earthquake to recur within the next 50 years.

2 Overview of the Meishan fault

In 1906, an $M_L = 7.1$ earthquake was occurring near Meishan Township in central Taiwan, killing about three thousand people at that time. Because the earthquake was near Meishan Township in central Taiwan, later the fault and the earthquake were named after the location as the Meishan fault and the Meishan earthquake, respectively. After the earthquake, field investigation showed that the surface rupture was around 15 km, with a fault scarp as large as 3 m (Lin et al., 2008).

Especially after the 1999 Chi-Chi earthquake, the Central Geological Survey Taiwan (CGST) has conducted a variety of investigations on the active faults in Taiwan. The investigations included field survey, geophysical testing, GPS monitoring, etc. After the investigation, the data such as earthquake return period and fault slip rate were published (Lin et al., 2008, 2009), and used in a variety of earthquake analyses later on. For example, Cheng et al. (2007) incorporated fault slip rates in their seismic hazard assessment.
analysis, and Wang and Wu (2014) used those earthquake magnitudes and return periods to conduct a risk assessment for active faults in Taiwan.

3 The Poisson process and earthquake probability

Given earthquake occurrences following the Poisson model, the probability for the next event to recur by time \( t^* \) is considered following the exponential distribution, with a cumulative density function (CDF) as follows (Ang and Tang, 2007):

\[
\Pr(T \leq t^*; \nu) = 1 - e^{-\nu t^*} \tag{1}
\]

where \( \nu \) is the mean rate. For example, given a return period of 162 years, the mean rate is equal to \( \frac{1}{162} \text{year}^{-1} \).

Using this model, we can estimate the earthquake probability associated with the Meishan fault. On the condition that the earthquake has not occurred by the end of 2013 since 1906 (the very last occurrence), the earthquake probability in 2014 is calculated as follows given \( \nu = \frac{1}{162} \text{year}^{-1} \):

\[
\Pr(\text{year 2013} < T \leq \text{year 2014}|T > \text{year 2013}) = \frac{\Pr(107 \text{years} < T \leq 108 \text{years})}{\Pr(T > 107 \text{years})} = \frac{\Pr(T \leq 108 \text{years}) - \Pr(T \leq 107 \text{years})}{1 - \Pr(T \leq 107 \text{years})} = 0.006 \tag{2}
\]

On the other hand, assuming the earthquake has not occurred by the end of 2023, the earthquake probability in year 2024 is:

\[
\Pr(\text{year 2023} < T \leq \text{year 2024}|T > \text{year 2023})
\]
\[
\frac{\Pr(117 \text{ years} < T \leq 118 \text{ years})}{\Pr(T > 117 \text{ years})} = \frac{\Pr(T \leq 118 \text{ years}) - \Pr(T \leq 117 \text{ years})}{1 - \Pr(T \leq 117 \text{ years})}
\]

\[
= \frac{(1 - e^{-118/162}) - (1 - e^{-117/162})}{1 - (1 - e^{-117/162})} = 0.006
\]

From the two calculations, we can see the “memory-less” effect of the stationary model: the time-invariant probability (i.e., 0.006) was only governed by the length of time (i.e., one-year interval), but irrelevant to the year either in 2014 or 2024. To demonstrate the stationary model more clearly, Fig. 1 shows the time-invariant probability for each year of the next 100 years, on the condition that the event has not occurred by the end of the previous year.

4 A new non-stationary model

4.1 Overview of Mohr–Coulomb failure criterion

The Mohr–Coulomb failure criterion is a model to describe a material’s mechanical behavior subject to external stresses (Pariseau, 2007), and it is commonly applied to rock mechanics, soil mechanics, etc. Figure 2 is a schematic diagram illustrating the essentials of the model. Basically, as the stresses represented by a Mohr circle that is below the failure envelope, a shear failure is not expected. By contrast, as long as the Mohr circle is in contact with the failure envelope, a shear failure can be expected in the material. Understandably, the failure envelope is governed by two strength parameters of the material, i.e., cohesion and friction angle.
4.2 Tectonic stress evolution between earthquakes

It is understood that the ongoing tectonic activities are the main reason for the increases in tectonic stresses with time, causing rock failures with the release of accumulated strain energy in a form of the so-called earthquake (Reid, 1910). With the theory, this section shows stress evolutions between two major earthquakes associated with a specific fault, in order to assess earthquake probabilities at a given time since the last recurrence.

After a major earthquake, two principle stresses in the vertical and horizontal directions should be adjusted to a similar level, governed by the coefficient of lateral earth pressure $K$ in rock. That is, the horizontal stress $\sigma_h$ at that time should be equal to $K \sigma_v$ (where $\sigma_v$ is vertical stress). Therefore at that time, the stresses can be represented like Mohr circle A in Fig. 2. Given the region subject to tectonic compression, the horizontal stress $\sigma_h$ could increase from Point A to Point B like Mohr circle B in Fig. 2, and at that time rock failure or earthquake is not expected. But as the compression continues, eventually the Mohr circle would be in contact with the failure envelope as $\sigma_1$ or $\sigma_h$ keeps increasing to Point C, and at that time a recurrence earthquake could be expected.

Based on the theory of the Mohr–Coulomb failure criterion, $\sigma_1$ at failure (Point C in Fig. 2) can be expressed as a function of $\sigma_3$ (vertical stress in this case) and two strength parameters of the shearing plane (or the fault plane), as follows (Pariseau, 2007):

$$\sigma_{1\text{ failure}} = \sigma_3 \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right) + \frac{2c \cdot \cos \phi}{1 - \sin \phi}$$  \hspace{1cm} (4)

where $c$ and $\phi$ are the cohesion and friction angle, respectively.

As a result, the earthquake probability in $t^*$ years after the last recurrence becomes a problem to estimate the likelihood whether $\sigma_1$ at that time is lower than $\sigma_{1\text{ failure}}$ or
not; therefore the earthquake probability can be formulated as follows:

\[ \Pr(\text{earthquake after } t^* \text{ years since last recurrence}) = 1 - \Pr(\sigma_{1,t^*} \leq \sigma_{1,\text{failure}}) \]  (5)

In this governing equation, \( \sigma_{1,t^*} \) and \( \sigma_{1,\text{failure}} \) are two random variables, and their formulations and calculations are detailed in the following.

### 4.3 Annual stress increment

To estimate \( \sigma_1 \) in \( t^* \) years after the last recurrence, we introduce annual stress increment (ASI) to formulate \( \sigma_1 \) that is a function of time:

\[ \sigma_{1,t^*} = \sigma_{1,t_0} + t^* \cdot \text{ASI} \]  (6)

where \( \sigma_{1,t_0} \) denotes the horizontal stress right after the major earthquake. Since it is equal to \( K \sigma_v \) as mentioned previously, the equation can be rewritten as:

\[ \sigma_{1,t^*} = K \sigma_v + t^* \cdot \text{ASI} \]  (7)

where vertical stress \( \sigma_v \) is associated with the weight of overburden rock above the failure plane, or fault plane, or the depth of earthquakes.

### 4.4 The mean and standard deviation of ASI

The next step of the earthquake probability assessment is to determine the mean value and standard deviation of annual stress increment, then to determine the probability distribution of the major principle stress (i.e., \( \sigma_1 \)) at a given time after the last occurrence. In the derivations, a boundary condition was considered: on a deterministic basis, the earthquake should recur when return period \( \tilde{t} \) is due, and on a probabilistic basis, the average value of \( \sigma_1 \) at return period \( \tilde{t} \) should be equal to \( \sigma_{1,\text{failure}} \). As a result,
the mean value of ASI (i.e., $\mu_{ASI}$) can be derived as follows:

$$E[\sigma_1 \cdot \tilde{t}] = E[K \sigma_v + \tilde{t} \cdot ASI] = \sigma_1_{failure} \Rightarrow E[ASI] = \frac{\sigma_1_{failure} - K \sigma_v}{\tilde{t}} = \mu_{ASI}$$

(8)

where $E[\cdot]$ denotes the mean value in the derivations.

By contrast, it is very difficult to characterize the standard deviation of ASI (denoted as $s_{ASI}$) without the studies on the region’s tectonic stress increment and its variability in time. Under the circumstances, our best estimates on the variability of ASI are from 0.25 to 1.0 in terms of COV (= standard deviation/mean value), and their influence on the earthquake probability assessments is discussed in the following with a parametric analysis.

As a result, given the COV of ASI equal to $n$, its standard deviation can be expressed with $n$ and its mean value (Eq. 8) as follows:

$$s_{ASI} = n \cdot \mu_{ASI} = n \cdot \frac{\sigma_1_{failure} - K \sigma_v}{\tilde{t}}$$

(9)

4.5 The mean and standard deviation of $\sigma_1$ at time $t^*$

With the derivations in Eqs. (6)–(9), we can estimate the mean and standard deviation of $\sigma_1$ at time $t^*$ since the last recurrence. Combining Eqs. (7) and (8), the mean value of $\sigma_1$ at time $t^*$ (denoted as $\mu_{\sigma_1 \cdot t^*}$) can be derived as follows:

$$\sigma_{1 \cdot t^*} = K \sigma_v + t^* \cdot ASI \Rightarrow E[\sigma_{1 \cdot t^*}] = E[K \sigma_v + t^* \cdot ASI] = K \sigma_v + t^* \cdot E[ASI]$$

$$= K \sigma_v + \frac{t^* \cdot (\sigma_1_{failure} - K \sigma_v)}{\tilde{t}}$$

(10)
Similarly, the standard deviation of $\sigma_1$ at time $t^*$ (denoted as $s_{\sigma_1,t^*}$) can be derived as follows with the standard deviation of ASI given in Eq. (9):

$$\sigma_{1,t^*} = K \sigma_v + t^* \cdot ASI$$

$$\Rightarrow V[\sigma_{1,t^*}] = V[K \sigma_v + t^* \cdot ASI] = t^{*2} \cdot V[ASI] = t^{*2} \cdot s_{ASI}^2$$

$$\Rightarrow s_{\sigma_{1,t^*}} = \sqrt{V[\sigma_{1,t^*}]} = t^* \cdot s_{ASI} = \frac{t^* \cdot n \cdot (\sigma_{1\text{-failure}} - K \sigma_v)}{\tilde{t}}$$

(11)

where $V[]$ denotes variance in the derivations, which is the square of standard deviation.

In addition to the mean value and standard deviation, the probability distribution is also needed for estimating the probability density function (PDF) of $\sigma_1$ at a given time. Without any reference available, the study presumes this variable follows the normal distribution, as a few probabilistic assessments adopted this model in the analyses when relevant information is not available (Abramson et al., 2002; Wang and Huang, 2013).

### 4.6 Summary of the methodology

Figure 3 is a schematic diagram summarizing the new framework for earthquake probability assessments. With the PDF of $\sigma_{1,t^*}$, the earthquake probability in $t^*$ years after the last occurrence, i.e., $1 - \Pr(\sigma_{1,t^*} < \sigma_{1\text{-failure}})$, can be simply calculated with the fundamentals of probability and statistics. To sum up, the new non-stationary analysis estimating recurrence earthquake probabilities associated with a specific fault is governed by a total of seven parameters, i.e., return period ($\tilde{t}$), rock strength parameters ($c$ and $\phi$), unit weight ($\gamma$), earthquake depth ($d$), coefficient of lateral earth pressure ($K$), and the variability of annual stress increment ($n$).

Table 1 summarizes the parameters used in this study, which are the best estimates from the literature except the variability of annual stress increment. For example, the unit weight of rock was considered from 25 to 30 kN m$^{-3}$ (Pariseau, 2007), and the
coefficient of lateral earth pressure in rock was considered from 0.2 to 0.5 (Gercek, 2007). As mentioned previously, given no references regarding the variability of annual stress increment around the study region, we presumed that it ranged from 0.25 to 1.0 as our best estimate in this study.

5 Case study: earthquake probability associated with the Meishan fault

With the new non-stationary model, a case study was presented in this section to estimate the recurring Meishan earthquake probabilities in each year of the next 100 years, on the condition that the major earthquake has not recurred by the end of the previous year.

On the use of those central values and \( n = 0.25 \) (see Table 1), Fig. 4 shows the earthquake probability for each year of the next 100 years, given that the earthquake has not occurred by the end of the previous year. The analysis shows that the non-stationary probability would increase from 0.0008 (in 2015) to 0.019 (in 2115) in the next 100 years, in contrast to a time-invariant probability of 0.006 from using the stationary Poisson model.

Figure 5 shows the earthquake probability on three scenarios with \( n = 0.25, 0.5 \) and 1.0. Based on the parametric study, the variability of annual stress increment could have a substantial influence on the earthquake probability. Figure 6 shows a schematic diagram to help explain such a variation. For the case with low ASI variability, the distribution is rather concentrated, and therefore the earthquake probability [i.e., \( 1 - \text{Pr}(\sigma_{1,t} < \sigma_{1\text{ failure}}) \)] is relatively small. By contrast, for the case with high ASI variability, the distribution of \( \sigma_{1,t} \) that is less concentrated could lead to a higher earthquake probability at that time.

It is worth noting that the result given in Figs. 4 and 5 is in terms of conditional probability rather than cumulative probability, and this is the reason why the curves are not necessarily increasing with time. Understandably, the purpose of using conditional...
probability is to compare the new analysis with the Poisson model on the same basis, in which the probability calculated refers to conditional probability.

6 Monte Carlo simulation

The estimates given in Figs. 4 and 5 were on a deterministic basis for using those central values in the analysis (see Table 1). In this section, we introduce a probabilistic analysis to estimate the earthquake probability, in which the parameters were considered as random variables that are uniformly distributed within the best-estimate range (see Table 1). Because the analyses would become much more complex and the analytical solution might not be available, we employed Monte Carlo Simulation (MCS) to solve the problem like many other probabilistic studies (e.g., Wang et al., 2010, 2012; Moghaddasia et al., 2011).

Basically, the MCS of this study is to generate random parameters (e.g., $n$, $\phi$, ...) within the best-estimate range at first, then substituting them in the governing equations of the non-stationary model to compute a random earthquake probability. With the randomization repeated for a number of times, a series of earthquake probabilities become available for calculating the mean probability or confidence interval, the fundamentals of Monte Carlo Simulation.

For example, Fig. 7 shows the MCS with a sample size of 10 000 to estimate the earthquake probability associated with the Meishan fault in years 2015 ∼ 2040, on the condition that the earthquake has not recurred by the end of year 2014 since the last occurrence in 1906. Accordingly, the earthquake probability within the next 25 years is about 0.15, with a 95 % confidence interval from 0.09 to 0.18. Besides, the simulation shows that the estimate in this case should be asymmetrically distributed, with a longer tail in the left-hand side of its probability density function.

Using the new non-stationary model and Monte Carlo Simulation, Fig. 8 shows another two analyses estimating the earthquake probabilities associated with the Meishan fault within the next 50 and 100 years. The result shows that the earthquake probability
in 2015 ~ 2065 is about 0.3 with a 95 % confidence interval from 0.26 to 0.36, and the probability could increase to 0.53 (with a 95 % confidence interval from 0.41 to 0.68), as far as a longer time interval of 100 years was concerned.

7 Discussions

7.1 Earthquake is stationary or non-stationary?

Although earthquake occurrence should be non-stationary from theory to experience, a recent statistical study somehow provided the support that earthquake occurrence should follow a stationary Poisson process by analyzing the seismicity around Taiwan (Wang et al., 2014). But when closely comparing the statistical study with this study, we can find that the problems targeted are different: this study is focusing on a non-stationary model associated with a specific fault, and the recent statistical analysis was to examine whether a regional seismicity should follow the Poisson distribution. Therefore, although using the Poisson distribution to model regional seismicity was supported with earthquake statistics, the conclusion is irrelevant to this study about a new non-stationary model for earthquake probability assessments as far as a given active fault is concerned, which should be increasing with time and should not be independent of the date of the last occurrence.

Figure 9 is a schematic diagram that helps explain the relationship between the two different problems. For each fault, the recurring earthquake should be a non-stationary process, with earthquake probability reset at the time while a major earthquake is recurring, then increasing with time until the next recurrence. By contrast, the seismicity in a region would become stationary after combining many non-stationary processes. Take Fig. 9 for example, the sum of the probability at \( T = t_0 \) should be similar to that at \( T = t_1 \), although the earthquake probability at \( T = t_0 \) could be very low for Fault D, in contrast to a very low probability at \( T = t_1 \) for Fault A.
The relationship can be simply explained with an analogy of a patron-and-bank problem: for each patron (analogy to each fault), going to the bank is a non-stationary process, with the probability increasing with time since the last visit. But for the bank (analogy to the seismicity), it is a stationary process with the number of patrons that is more or less the same in any time, after combining that many non-stationary processes from each patron.

7.2 Earthquake is very difficult to predict

Given the recent major earthquakes unpredicted, indeed earthquake prediction is very challenging with our limited understandings of the nature. As a result, like many earthquake studies, this study is to propose a new perspective in earthquake occurrence analyses, but not to claim it is a perfect solution to earthquake prediction. However, it must be noted that the motivation and contribution of this study is to analyze the non-stationary earthquake occurrence with a new non-stationary model, an improvement over the use of the Poisson calculation in earthquake probability assessment, and the key scope of this study.

8 Conclusions

This study presents a new non-stationary analysis for earthquake probability assessments associated with a specific fault. The basics of the analysis are to estimate the probability distribution of the shear stress on the fault plane at a given time, then comparing it to the failure stress.

In addition to the methodology, this paper also presents a case study to estimate the earthquake probability associated with the Meishan earthquake in central Taiwan, a growing concern to the local community because of a short earthquake return period reported. In contrast to a stationary probability of 0.006 from the Poisson calculation, the non-stationary analysis estimates the earthquake probability should increase from
0.0008 (in 2015) to 0.019 (in 2115) in the next 100 years, on the condition that the earthquake has not recurred by the end of the previous year.

In addition, given the recurring Meishan earthquake has not occurred by the end of 2014, the earthquake probability in the next 50 years is about 0.3 (with a 95% confidence interval from 0.26 to 0.36), on the basis of the new non-stationary model and a probabilistic analysis using Monte Carlo Simulation.

Acknowledgements. We appreciate the valuable comments from Editor D. Keefer, making the submission much improved in so many aspects.

References


Table 1. Summary of the model parameters used in the analyses.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Earthquake depth (km)</th>
<th>Unit weight (kN m$^{-3}$)</th>
<th>Cohesion (MN m$^2$)</th>
<th>Friction angle (degrees)</th>
<th>Return period (years)</th>
<th>$K^a$</th>
<th>$n^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>5 ~ 15</td>
<td>25 ~ 30</td>
<td>3.6 ~ 22.7</td>
<td>22 ~ 46</td>
<td>–</td>
<td>0.2 ~ 0.5</td>
<td>0.25 ~ 1.0</td>
</tr>
<tr>
<td>Central value</td>
<td>10</td>
<td>27.5</td>
<td>13.2</td>
<td>34</td>
<td>162</td>
<td>0.35</td>
<td>–</td>
</tr>
</tbody>
</table>

$^a K$ = the coefficient of lateral earth pressure in rock; $^b n$ = the coefficient of variation for annual stress increment.
The Poisson calculation for earthquake probability assessment given a one-year interval and a mean annual rate of $1/162$ per year.

Figure 1. The time-invariant, stationary earthquake probability estimated with the Poisson calculation, given the best-estimate mean rate $= 1/162 \text{ year}^{-1}$.
Figure 2. Schematic diagram illustrating the Mohr circles and the Mohr–Coulomb failure criterion applied to this study.
The distribution of stress at time $t^*$ after the last occurrence
Earthquake probability at time $t^*$ after the last occurrence

Figure 3. Schematic diagram illustrating the essentials of the new model, by developing the probability distribution of $\sigma_1$ at time $t^*$ after the last occurrence.
Figure 4. The earthquake probability associated with the Meishan fault in central Taiwan estimated with the new non-stationary model.
Figure 5. The earthquake probability associated with the Meishan fault on three different scenarios in annual stress increment.
Figure 6. Schematic diagram illustrating the influence of annual stress increment on earthquake probability assessments.
Figure 7. The Monte Carlo Simulation to estimate the earthquake probability with the considerations of the range of the input data (see Table 1).
Figure 8. The 95% confidence interval of the earthquake probability given three different periods of time, on the condition that the earthquake has not occurred by the end of 2014.
Figure 9. Schematic diagram illustrating the stationary process after combining a number of non-stationary processes; at $T = t_0$ and $T = t_1$ for example, the sum of the probabilities would be similar, although the probability is very low for Fault D at $T = t_0$, and it is very low for Fault A at $T = t_1$. 