Flood Frequency Analysis supported by the largest historical flood.

W. G. Strupczewski¹, K. Kochanek¹ & E. Bogdanowicz²

1) Institute of Geophysics Polish Academy of Sciences, Ksiecia Janusza 64; 01-452 Warsaw, Poland,
e-mails: wgs@igf.edu.pl, kochanek@igf.edu.pl
2) Institute of Meteorology and Water Management, Podlesna 61, 01-673 Warsaw, Poland,
e-mail: ewa.bogdanowicz@imgw.pl

ABSTRACT

The use of non-systematic flood data for statistical purposes depends on reliability of assessment both flood magnitudes and their return period. The earliest known extreme flood year is usually the beginning of the historical record. Even if one properly assess the magnitudes of historic floods, the problem of their return periods remains unsolved. The matter in hand is that only the largest flood (XM) is known during whole historical period and its occurrence marks the beginning of the historical period and defines its length (L). It is common practice to use the earliest known flood year as the beginning of the record. It means that the L value selected is an empirical estimate of the lower bound on the effective historical length M. The estimation of the return period of XM based on its occurrence (L), i.e. \( \hat{M} = \frac{L}{2} \), gives severe upward bias. The problem arises that to estimate the time period (M) representative of the largest observed flood XM.

From the discrete uniform distribution with support 1, 2, ..., M of the probability of the L position of XM one gets \( \hat{L} = \frac{M}{2} \).

Therefore \( \hat{M} = 2\hat{L} \) has been taken as the return period of XM and as the effective historical record length as well this time.

As in the systematic period (N) all its elements are smaller than XM, one can get \( \hat{M} = 2\left(\frac{L + N}{2}\right) \).

The efficiency of using the largest historical flood (XM) for large quantile estimation (i.e., one with return period \( T = 100 \) years) has been assessed using the ML method with various length of systematic record (N) and various estimates of historical period length \( \hat{M} \) comparing accuracy with the case when systematic records alone (N) are used only. The simulation procedure used for the purpose incorporates N systematic record and one largest historic flood (XM) in the period \( \hat{M} \) which appeared in the \( \hat{L} \) year backward from the end of historical period. The simulation results for selected distributions, values of their parameters, different N and M values are presented in terms of bias and RMSE of the quantile of interest are more widely discussed.

Keywords: Flood frequency analysis; Historical information; Error analysis, Maximum Likelihood, Monte Carlo simulations.

1. INTRODUCTION

Flood engineering usually deals with the determination of the flood of a given return period \( T \) years, i.e. the flood quantile \( X_T \) or the design flood. The problems with the assessment of these parameters result from short time series (\( N < T \)), unknown probability distribution function of annual peaks, error corrupted data, the simplifying assumptions as of identical independently distributed (i.i.d.) data and, in particular, the assumption of stationarity of relatively long data series. All these account for high uncertainty of the upper quantile estimate. The effect of sample size is widely documented for various distribution models and estimation methods, thus, it is obvious that due to a short sample the confidence interval of the design flood estimate is already very broad. In addition to Flood Frequency Analysis (FFA) other sources of error would result in increasing uncertainty in the design flood estimate. This feature is not appreciated by the designers as they want to have only one value for designing flood related structures. Conversely, efforts to improve the accuracy of estimates of the hydrologic design value by specifying the various sources of uncertainty and incorporating them in the analysis produce the opposite effect from the one intended.

To improve the accuracy of estimates of upper quantiles all possible sources of additional information and ‘statistical tricks’ are used, such as: independent peaks above the threshold, seasonal approach, regional analysis, record augmentation by correlation with longer nearby records and, finally, augmentation of the systematic records by historical and paleo-flood data.

Frequency analysis of flood data arising from systematic, historical, and paleo-flood records has been proposed by several investigators (a review Stedinger and Baker, 1987, Frances et al. 1994, MacDonald, 2013). The use of non-systematic flood data for statistical purposes depends on reliability of assessment both flood magnitudes and their return period. If the historical record is available, the information about the floods larger than prevailing majority of floods reported in the systematic record can be introduced to the datasets and, if we
are lucky, the unique information about the largest reported floods. Serious difficulties relate to the
(un)availability and (not-) exhaustiveness of historical information, the (low) quality and (in)accuracy of
historical sources. As if it was not enough, depending on the number of parameters and their method of
estimation, the estimates of high quantities are more or less sensitive to the largest observed floods.

The earliest and simplest procedures for employing historical and pale-flood data were based on plotting
1985, Cohn, 1986). The PWM method and L-moment method were introduced by Ding & Yang (1988), Wang
(1990, 1996) and Hosking (1995). To deal with historical and pale-floods Hosking and Wallis (1986 a, b)

applied the maximum likelihood (ML) as the estimation method. Recently the Bayesian estimation paradigm has
been incorporated (Vigilione et al., 2013, Parent and Bernier, 2003, Reis and Stedinger, 2005). It takes into
account that the historical floods are known with uncertainty, for instance with lower and upper bounds (in fact
the effect of corrupted historical flood magnitudes was investigated by Hosking and Wallis via MLE mentioned
as early as in 1986 a, b) The subject of historical floods currently constitutes one of the main scientific threads in
flood frequency analysis (MacDonald, 2013, Payrastre et al., 2011, 2013). It is important to add, that the
inclusion of historical information is recommended in a number of national and international policy documents
e.g. EU Flood Directive. The log Gumbel, Weibull and Gamma distributions together with maximum likelihood
method were considered by Frances et al. (1994) to tackle systematic and historical or pale-flood data in FFA.
To assess the potential statistical derived from historical information the asymptotic variances of the quantile
estimates from the systematic records alone and the combined time-series were compared by means of computer
simulation experiments. The study performed to define the length (M) of historical period indicate that value of
historical data for estimating flood quantiles can vary depending on only three factors: the relative magnitudes of
the length of the systematic record (N) and the length of the historical period (M); the return period (T) of the
flood quantile of interest; and the probability threshold defining the historical floods.

Most often it is the first historical large flood that is the most remembered (and described in historical
sources) and, therefore, it is usually not considered as important (or simply not known) what had happened
before (Girgus and Strupczewski, 1965). In other words, only the largest (paleo-)historical flood is usually
known for either it was best remembered (and thus recorded) because of its destructive character and taking a
toll on many lives or, in case of pre-historical time, the largest inundation swept away any evidence of smaller
floods that occurred earlier. The date of the first recorded historical flood is taken as the historical memory
length L, i.e. L becomes the duration of non-systematic period commencing on the large flood. Even if one
properly assess the magnitudes of historic floods, the problem of their return periods remains unsolved. In most
literature examples (specially Benson, 1950, Dalrymple, 1960, IACWD, 1982, Zhang, 1982 and NERC, 1975,
p.177) one reads that effective length of historical record M used for frequency analysis is always taken to be the
period from the first extraordinary flood to the beginning of the systematic record, i.e. L.

The matter in hand is that only the largest flood (XM) is known during the whole historical period and its
occurrence marks the beginning of the historical period and defines its length (L) (Fig.1). That is because the
beginning of the historical period was somehow forced by the appearance of the largest flood (XM) but in fact its
unusual magnitude corresponds rather to a longer return period than L (or, if in systematic record all
observations are smaller than XM, to (L+N)-period) , i.e. the probability that the actual return period of XM is
longer than the L is greater than fifty percent.

Attempts to eliminate or lessen this error lead us to the estimation the time period (M) representative of the
largest observed flood XM as accurately as possible. In order to do so, we will carry out the evaluation of the
efficiency of using the largest historical flood (XM) for large quantile estimation and its comparison with the
case when systematic records alone (N) are used. To keep and preserve the unspoiled genuine information
contained in the observation (XM, L), the return period \( \hat{M} \) of the largest observed historical flood (XM) should
be assessed without data from the systematic record providing that it does not contain elements larger than XM
values.

It is obvious that the return period of the historical flood assessed on the base of the year of occurrence (L)
represents just the lower limit of its real empirical return period (M). Of course, there is an upper empirical limit
as well, which however, can not be estimated unambiguously. This is so because, if the occurrence of a large
flood was reported in a given year, for sure a similar or more serious flood a year before would have been also
noted and commented in historical sources (Hirsch and Stedinger, 1987). The same can be stated for horizon of
two, three, four, etc. years. If we could identify this time span, we would have determined the upper limit of the
empirical return period.

The estimation of M based on the date of the first extraordinary flood occurrence exacerbates an already
severe imprecision. By defining as historical floods all floods during the M period above a given threshold and
taking four different plotting position formulas, Hirsch and Stedinger (1987) calculated (with the use of Monte
Carlo experiment) the magnitude of the upward bias of the plotting position of the largest sample elements
occurring when L is taken as the beginning of the historical record. Doing so they noticed that L is a random
variable dependent on the flood-producing process itself; this would be a violation of the assumption of the
plotting position formulas.

Similarly, Hosking and Wallis (1986 a, b) use Monte Carlo (MC) computer simulation to assess whether a
single paleo-flood estimate, when included in a single-site Maximum Likelihood (ML) flood frequency analysis
procedures, gives a worthwhile increase in the accuracy of estimates of extreme floods. They found that the main
factors affecting the utility of this kind of palaeological information are the specification of the fitted flood
frequency (whether it has two or three unknown parameters) and the size of the measurement error of paleo-
discharge estimates. Errors in estimating the date of the paleo-flood are considered to be of minor importance.
For distributions with higher CV or skewness the difference between the effects of the errors of the magnitude of
paleo-flood and its return period is smaller.

Note that the randomness of the systematic records time series of i.i.d. variable can also be sometimes
questioned and undermined, e.g. when the largest value XM of a time series intentionally terminates the N-
elements’ systematic record. Then the XM is the last element of the N-element time-series. Such a case may arise
when a water gauge was swept away by a heavy flood (XM) and not restored, or the intentional movement of the
hydrological station. As before, the use of such a series in FFA with \( \hat{M} = N \) will lead to an overestimation of
large quantiles.

2. PROBLEM FORMULATION

The object of the paper is to assess by use of the maximum likelihood (ML) method whether there is any impact
of the largest flood terminating the time series assuming its magnitude (XM) is known. Therefore, the case of
systematic data completed by largest flood is compared with the case where records contain systematic data
only. These two variants are examined by comparing the bias (B) and the root mean square error (RMSE) of
flood quantiles. The two two-parameter distributions, namely Gumbel and Weibull were used when applying the
simulation experiments. The emphasis is put on the effect of misspecification of the return period (M) of the
largest historical (paleo-) flood (XM) and on the proper assessment of the M estimate on the basis of XM
occurrence (L). So far, the results of such research has not been presented in the hydrological literature.

The theoretical framework of our research is based on Maximum Likelihood estimation which has been
generally found to have desirable properties for combine systematic and historical information (Frances et al.,
1994, Stedinger and Cohn, 1986, Naulet et al., 2005). It is assumed that the annual maximum floods are
independent and identically distributed.

Assessment of the return period M of the XM flood

Hirsch and Stedinger (1987) considered that the time of occurrence of the earliest documented historical flood L
is the random variable defining a lower bound of the sample size used for computation of plotting positions. The
position L of the largest in M period element (XM) (Fig. 1) is the random variable being discretely uniformly
distributed in the M period, i.e. \( p_t = 1/M \) for \( t = 1, 2…M \). Obviously the magnitude of the largest element (XM) is
also a random variable. It can correspond in the population to a smaller or larger value of the exceedance
probability than \( 1/M \) defining the effective return period (Mg) of XM. Therefore the difference (Mg – L) is not
restricted in sign.

Assume that the return interval (M) of XM is known. As L is uniformly distributed variable in the M length
time series with support \( L \in [0,1,…,M] \), one gets \( E(L) = M/2 \) and \( V(L) = M^2/12 \). In reality \( M \) is not known and its
assessment is our goal. Taking the observed L value as the estimate of the expecting value, i.e. \( L = E(L) \) we get
the M estimate equal \( \hat{M} = 2L \). Because regardless of the estimation method the quantile estimators are not in
general linear function of \( \hat{M} \), the minimum bias of quantile \( B (\hat{x}_p) = E [\hat{x}_p (\hat{M}) - x_p] \) does not necessarily
correspond to the zero-bias of \( \hat{M} \), i.e. to \( \hat{M} = 2L \). If in the systematic period (N) all its elements are smaller
than XM, one can get \( \hat{M} = 2(L + N) \). Note that usually \( N << L \).

3. SIMULATION PROCEDURE

The simulation procedure incorporates N systematic record and one largest historic flood (XM) in the period \( M \)
which appeared in the L year backward from the end of historical period (Fig. 1). Obviously, the systematic
record and both magnitude (XM) and year of occurrence (L) randomly vary from simulation to simulation. As an
estimate of the length of the historical period shall be successively \( \hat{M} = L, 2L \) and the actual value \( \hat{M} = M \), i.e.
the length of the period M in simulation experiment.
First, generate a gauged record $x_1, x_2, \ldots, x_N$ of independent random variates from the assumed (two-parameter) flood-like distribution $[F(x)]$ with parameters chosen to give specified values of CV. Then generate historical series of the same distribution of the length $M$, i.e. $y_1, y_2, \ldots, y_M$, and find the maximum event $(XM)$ of the historical series denoting the time $(L)$ of its occurrence. Since the random variables $(XM)$ and $L$ are mutually independent the $XM$ can be generated from the distribution of the largest element in a $M$-element series, i.e. $F(M) = F_1(M) = F^M(y)$, while the corresponding time of its occurrence $(L)$ from the discrete uniform distribution with support $(1, 2, \ldots, M)$.

A flood frequency distribution fitted by the method of maximum likelihood has a distribution function $F(x; \theta)$ and a density function $f(x; \theta)$, where $\theta$ is a vector of unknown parameters, then the likelihood function $(L)$ is taken to be

$$L(\theta; x, y) = F_x^{M-1}(y = XM; \theta) \cdot f_x(y = XM; \theta) \cdot \prod_{i=1}^{N} f_x(x_i; \theta),$$

i.e., the use of incomplete data likelihood, where $\hat{M} = L, 2L$ and $M$ and for systematic record only

$$L(\theta; x) = \prod_{i=1}^{N} f_x(x_i; \theta).$$

Calculate quantile estimates $\hat{X}_T = F^{-1}(1-1/T; \hat{\theta})$ for $\hat{M} = L, 2L$ and $M$ and the systematic record $(N)$ only (i.e. when $\hat{M} = 0$), where $F^{-1}$ is the inverse distribution function of the fitted flood frequency distribution, $\hat{\theta}$ is the maximum likelihood estimate of $\theta$, and $T$ is the return period of interest.

Repeat the above steps a large number of times $(i)$ and calculate the mean and variance of $\hat{X}_T$, and hence the relative bias $RB$ and relative $RMSE$ of $\hat{X}_T$ taking $\hat{M}_i = L_i, 2L_i$ and $M$ and the systematic record $(N)$ only $(\hat{M} = 0)$ considered as an estimator of the true quantile $X_T = F^{-1}(1-1/T; \theta)$. If in a generated series one gets max$(x_1, x_2, \ldots, x_N) \geq XM$ such simulation is ignored which allows us to assume $\hat{M} = 2L$.

4. SIMULATION RESULTS

The concise frame of this paper made us to limit the number of models we took into consideration in our calculations. In order to lessen the number of the figures for all the combinations of CS and CV values we resigned from three-parameter distributions such as generalised extreme value (GEV) and turned into its two-parameter special forms, namely Gumbel (Gu) and Weibull (We). Another cause was also that, however theoretically sound, the GEV working perfectly for large samples often fails in far-from-asymptotic samples which we examine in this study. We scrutinised a number of two- and three-parameter distribution functions in terms of their best fit to hydrological annual and seasonal peak flows in Poland and it turned out that despite the regime of the river other models were preferred rather than GEV (Strupczewski et al, 2012, Kochanek et al, 2012). However, the crucial argument after the choice of the parent distribution was the pioneering works of Frances et al. (1994) that we wanted to continue and develop. Results of simulation experiments are shown for Gu and We distributions with four values of the coefficient of variation $CV = 0.25, 0.5, 0.75, 1.0$, with two different lengths of systematic records $N = 15, 50$ and the length of effective historical period $M = N \exp(a)$ where $a \in [0,3]$. Due to the limited capacity of this paper without the loss of generality, only the selected results were presented in Figs. 2-5, namely for $CV=0.25$ and 1.0; the results for $CV=0.5$ and 0.75 locate themselves between those presented in the figures. Results for the correct value of the return period $(\hat{M} = M)$ are compared with those got for $\hat{M} = L, 2L_i$. For completion the results for the systematic record only (i.e. $\hat{M} = 0$) were presented in all figures (solid line). Of course, for this case the results do not depend on $M$ and in consequence on $\log(M/N)$.

5. DISCUSSION OF THE RESULTS

- The shorter the gauged record $(N)$ is, the more useful the historical information.
- Using as the estimate of the true return period of largest historical flood $(XM)$ the historical memory length $(L)$ results in considerable upward bias $RB$ of 1% quantile far exceeding the bias for the systematic record only. Its value increases with $CV_i$ (and $CS_i$) and with the $MM/N$ ratio.
- Using in ML estimation the $\hat{M} = 2L$ instead of $\hat{M} = L$ considerably reduces the bias and further reduction is obtained for the $\hat{M} = M$, i.e. for the return period $(M)$ of the largest historical flood $XM$. 

4
Although the use of \( \hat{M} = 2L \) instead of \( \hat{M} = L \) reduces the bias more than twice, it is still \textit{circa} 40% larger than the bias of a known return period \( M \) of \( XM \), and comparable or lower than the bias from systematic record \( (N) \).

As far as the relative root mean square error (RRMSE) of 1% quantile is concerned, for both Gumbel and Weibull models one can notice some reduction in its values when one uses \( L, 2L \) or \( M \) return periods in comparison to the systematic sample. The worst reduction of RRMSE one gets for \( L \), better for \( 2L \) and the best for \( M \) which means that it is worth, at least, considering using a historical measurement \( XM \) in upper quantile estimation and then set the return period of \( XM \) to \( 2L \) rather than \( L \) if we do not know \( M \).

The reduction in RMSE for both models (Gumbel and Weibull) rises generally with \( M/N \) ratio. In other words: the bigger \( M \) (compared to \( N \)), the higher distance between RRMSE values got for the sample with additional historical information and the systematic series. It goes without saying, that for \( N = 15 \) one gets better reduction than for \( N = 50 \).

For the Gumbel model, regardless of the sample return period, \( L, 2L \) or \( M \), the relative reduction in RRMSE compared to systematic samples does not depend on \( CV \). It does not hold for Weibull where the reduction decreases with \( CV \), e.g. between \( CV = 0.25 \) and 1.0 there is usually a few-percent difference which is minimal (almost marginal) for \( \hat{M} = M \).

For Gumbel model reduction in comparison to systematic sample for \( \log(M/N) = 3 \), \( CV = 0.25 \) and \( N = 15 \) the reduction gets up to 2.2, 3.6 and 5.3% for \( L, 2L \) and \( M \) respectively. For \( N = 50 \) these numbers are roughly three times smaller.

For Weibull the gain in RRMSE is more spectacular and for \( \log(M/N) = 3 \), \( N = 15 \) and \( CV = 0.25 \) equals to 3.4, 4 and 4.9% for \( L, 2L \) and \( M \) respectively (when \( CV = 1.0 \) the gain is c.a. four times lower). For \( N = 50 \) the general trend for Weibull remains the same as for \( N = 15 \) but the reduction of RRMSE is smaller.

To sum up the RRMSE issues, the inclusion of the largest historical flood in FFA with \( \hat{M} = 2L \) (i.e. the effective historical record length) gives a few-percent reduction in RRMSE of extreme flood estimates. However, the reduction is \textit{circa} 20 up to 60% lower than if we took \( M \) as the length of simulation period. The true value of \( M \) is not available in reality, so one is doomed to use \( 2L \) instead.

Therefore, to benefit from the largest historical observation every effort should be made to establish \( M \) accurately.

In the absence of any information about the period preceding the occurrence of \( XM \) one should put \( \hat{M} \) equal to \( 2L \) or \( 2(L+N) \).

The benefit from including the largest historical flood of a given value is measured by the reduction of RRMSE. It depends on:

- the length of systematic record \( (N) \),
- the ratio of the true return period of \( XM \), i.e. \( M \) to \( N \),
- the ratio of \( N \) to the return period of quantile of interest,
- the \( CV \) and skewness of the parent distribution.

### 6. CONCLUSIONS

Errors in historical data reduce, of course, the utility of the data for improvement of the estimation of flood magnitude at a given return period. In the simulations (Figs. 2-5) it was assumed that the magnitude of the largest historical flood \( (XM) \) was measured without error and the same was assumed for the systematic record. It is realistic to suppose that the \( XM \) flood was measured much less accurately than the gauged record. Error in estimating the largest historical magnitude \( (XM) \) is much more important than error in estimating the date of its occurrence (\textit{e.g.} \textit{Hosking and Wallis}, 1986 a, b). It is significant that inspired by the practice of efforts to improve the accuracy of estimates of flood quantiles through more realistic assumptions and a fuller use of the information they give just the opposite effect leads to increased uncertainty of flood estimates.

The next step should be to refer to the general problem of historical information when the applied distribution model is false, which is always the case (\textit{Strupczewski et al.}, 2002). On the other hand, the uncertainty of the paleo-historical floods (both in terms of their magnitude and return period) combined with considerable increases in the complexity of the problem (when compared to analysis of systematic data only) provokes a fundamental question, whether the whole operation is worth a candle. Therefore, whether to include the paleo-historical information or turn a blind eye to it, is a matter of conscience.

All these generate two important practical problems which we leave for further study, namely:

1. What is the theoretical upper limit of accuracy of high quantile estimation when the theoretical value (i.e. taken from the parent distribution) of return period for \( XM \) is known?

2. Here in our simulation experiment we assumed the knowledge of the true (parent) distribution function. The role of historical information when the assumed distribution serves as the model of the true distribution remains, for the time being, unknown.
Only the solutions to these two problems completed by the consideration of the observation errors in FFA brings us closer to the answer to the fundamental question stated above, i.e. whether the available paleo-historical record can give worthwhile improvement in flood estimates.

Acknowledgements. This research project was partly financed by the grant of the Polish National Science Centre titled ‘Modern statistical models for analysis of flood frequency and features of flood waves’, contract nr UMO-2012/05/B/ST10/00482 and made as the Polish voluntary contribution to COST Action ES0901 ‘European Procedures for Flood Frequency Estimation (FloodFreq)’.

8. REFERENCES


Payrastre, O., Gaume, E. and Herve, A.: Historical information and flood frequency analyses: which optimal features for historical floods inventories? Houille blanche-revue international de l’eau, Issue: 3 Pages: 5-11 DOI: 10.1051/hhb/2013019, published: JUN 2013


List of figures:

Figure 1. The case of N systematic and one largest flood in the beginning of historical period.

Figure 2. Relative bias (RB) and relative root mean square error (RRMSE) of $\hat{X}_{T=100}$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M}_i = 0, L, 2L, M$. Parent distribution Gumbel with CV equal: 0.25 and 1.0 and $N=15$. Fitted distribution Gumbel.

Figure 3. RB and RRMSE of $\hat{X}_{T=100}$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M}_i = 0, L, 2L, M$. Parent distribution Gumbel with CV equal 0.25 and 1.0 and $N=50$. Fitted distribution Gumbel.

Figure 4. RB and RRMSE of $\hat{X}_{T=100}$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M}_i = 0, L, 2L, M$. Parent distribution Weibull with CV equal 0.25 and 1.0 and $N=15$. Fitted distribution Weibull.

Figure 5. RB and RRMSE of $\hat{X}_{T=100}$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M}_i = 0, L, 2L, M$. Parent distribution Weibull with CV equal 0.25 and 1.0 and $N=50$. Fitted distribution Weibull.
Figure 1. The case of N systematic and one largest flood in the beginning of historical period.
Figure 2. Relative bias (RB) and relative root mean square error (RRMSE) of $\hat{\mathbf{X}}_{T_{100}}$ as a function of gauge record length $N$ and historic period $M_i = 0, L_i, 2L_i, M_i$. Parent distribution Gumbel with CV equal: 0.25 and 1.0 and $N=15$. Fitted distribution Gumbel.
Figure 3. RB and RRMSE of $\hat{T}_\tau$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M}_a = 0, L_i, 2L_i, M$. Parent distribution Gumbel with CV equal 0.25 and 1.0 and $N=50$. Fitted distribution Gumbel.
Figure 4. RB and RRMSE of $\hat{T}_{X,100}$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M}_i = 0, L_i, 2L_i, M$. Parent distribution Weibull with $CV$ equal 0.25 and 1.0 and $N=15$. Fitted distribution Weibull.
Figure 5. RB and RRMSE of $\hat{T}_{100}$ as a function of gauge record length $N$ and historic period $M$ for $\hat{M} = 0, L_1, 2L_1, M$. Parent distribution Weibull with CV equal 0.25 and 1.0 and $N=50$. Fitted distribution Weibull.