Safe-Economical Route Model of a Ship to Avoid Tropical Cyclones Using Dynamic Forecast Environment

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Abstract

In heavy sea conditions related to tropical cyclones (TCs), losses to shipping caused by capsizing are greater than other kinds of accidents. Therefore, it is important to consider capsizing risk in the algorithms used to generate safe-economic routes that avoid tropical cyclones (RATC). A safe-economic routing and assessment model for RATC, based on a dynamic forecasting environment, is presented in this paper. In the proposed model, a ship's risk is quantified using its capsizing probability caused by heavy wave conditions. Forecasting errors in the numerical models are considered according to their distribution characteristics. A case study shows that: the economic cost of RATCs is associated not only to the ship's speed, but also to the acceptable capsizing probability which is related with the ship's characteristic and the loading condition. Case study results demonstrate that the optimal routes obtained from the model proposed in this paper are superior to those produced by traditional methods.

1 Introduction

Weather hazards are the main threat to shipping. The goal of weather routing is to plan routes that avoid weather hazards safely and economically. Tropical Cyclones (TCs), as a kind of hazardous weather, cause extensive damage to the ship and crew. The total loss caused by ship capsizing is very serious to the ship company and cargo owner. Compared to the capsizing, the loss of cargo and ship damage caused by other accidents is much less in TCs. Ships can avoid TCs safely with routes based on the methods applied in navigation practice –Sector diagram typhoon avoidance method (Chen,2004); 34KT rule (Holweg,2000); and the Diagram of the 1-2-3 rule (Wisniewski et al., 2009) – but routes generated by these methods

ignore costs and increase shipping expenses. Furthermore, in these models the ship's performance in resisting wind and waves is not considered.

The resolution and precision of ocean and atmospheric models are increasing in tandem with the rapid development of computational power, thus allowing more possibilities for ship routing with precise and less uncertain forecast results (Delitala et al., 2010). Many researchers have done weather routing work using ocean and weather numerical models, but the forecast errors are not considered in their work (Padhy et al., 2008; Maki et al.,2011). Nowadays, it is feasible to find routes to avoid TCs (RATC) using dynamic wind and wave fields as forecasted by numerical models. A minimum economic cost route was designed to avoid TCs from the following aspects: (1) The forecast errors of models are considered in the route design; (2) The ship's characteristics are considered in assessing the risk in the heavy weather conditions; (3) The ship's risk is quantified; (4) The ship's speed loss is considered.

2. Literature review

As a major transportation mode, maritime transportation play an important role in the international trade. The daily operating costs of a ship can be tens of thousands of dollars. Ship routing draws more and more attention from scientists and engineers. Last decade, there are lots of research about ship routing and scheduling. Linear programming model (LP), mixed integer programming (MIP) model and set partitioning (SP) formulation are used to solve the routing problem. Christiansen et al. (2004) gave a detail review of the past decade research about the ship routing and scheduling. In the research about optimization, lots of method are used to solve the problem, i.e. dijkstra's algorithm, semi-Markov decision process (Azaron and Kianfar, 2003) and so on. Weather routing is a ship routing problem with multiple restrictions and changing environment condition. The relevant literature includes research on weather routing, numerical forecast error, RATC, and vessel risk analysis.

There are several widely used methods for weather routing. These methods are based on the isochrone method and have been refined since its introduction. James (1957) proposed an isochrone method, which was widely used for decades. Based on the work of James, improvements have been made to update the method (Hagiwara,1989). Chen (1978) developed a dynamic programming algorithm to solve the minimum voyage cost problem under uncertainty constraints. In the algorithm, the sea-keeping features of the ship are a function of weather. McCord et al. (1999) investigated the potential for strategic ship routing through dynamic currents to determine 3-day routes. The results showed that this kind of strategic ship routing can reduce fuel consumption by 25% on average. Delitala et al. (2010) showed that weather routing improves ship performance by 37% thus supporting ship captains throughout an entire voyage. Panigrahi et al.(2008) and Padhy et al(2008) optimized the ship's route based on the output of the WAM (WAve Modelling). Maki et al. (2011) designed a real-coded genetic algorithm weather routing method to calculate the safety ratio and fuel

efficiency based on the probability of accidents caused by parametric rolling. Soda et al. (2011) gathered and used high-resolution wind and wave data forecasted using SWAN (Simulating Waves Nearshore model) and WRF (Weather Research and Forecasting Model) to study wave and wind effects on ship's manoeuvring. From the aspect of minimum pollute probability to the coast, fairways in the Gulf of Finland are designed based on the current condition from the ocean model (Andrejev et al. 2011).

The weather routing problem is complicated since weather conditions are uncertain. The forecast error in numerical models must be considered when designing weather routing (Magirou and Psaraftis, 1992). Hopkins (1997) examined the offshore forecast statistical errors derived from the numerical forecasting models. RMS (root mean square) errors in wind speed and wave height forecasts have a seasonal variation. The RMS errors of the WaveWatchⅢ model against altimeter and buoy data are 15% of the mean observed wave heights for most of the global domain (Tolman,2002). Bedard(2008) evaluated the forecast technology of WaveWatchIII based on the comparison with buoy-measured wave data in deep water of Washington and Oregon, U.S.A. Chu et al. (2004) compared the model result of WaveWatchIII with the T/P (Topex/Poseidon) significant wave height (SWH) data over the satellite crossover points in the South China Sea (SCS). The results showed that the model errors of SWH had Gaussian-type distribution. The research of Guo and Hou(2010) coincides with Chu's (Chu et al.,2004) research results in that the WaveWatchIII forecast error of SWH follows a Gaussian distribution.

Although the literature on weather routing is rich, little work focuses on RATC. The sector diagram typhoon avoidance method (Chen,2004), and the 34KT rule (Holweg, 2000) are widely used in navigation. Wisniewski (2009) discussed the application of the 1-2-3 rule for calculations for routing vessels using evolutionary algorithms. Liu et al. (2006) built a safety-economic decision-making model of RATC using risk theory and fuzzy information optimization. Zhang et al. (2010) proposed a multilevel decision method for RATC using multi-source track forecasting. Wu et al. (2010) built a model to evaluate the benefit for RATC. Wisniewski and Kaczmarek (2012) analysed reactions, such as reducing speed and changing course, when a ship is avoiding a TC. At present, these RATC methods are mainly based on TC forecast tracking, experience, and fuzzy analysis. All these methods, in the final instance rely on qualitative judgement. These methods also do not consider the performance or reactions of different vessels under the same wave or wind conditions. Therefore, different vessels might require different routing strategies to safely and economically avoid TC.

The quantification of risk to ships under heavy weather conditions is an important problem for weather routing. Most researchers assess a ship's risk using fuzzy analysis, the risk cannot be quantified (Zhang et al., 2010; Liu et al., 2006). In heavy sea conditions related to tropical cyclones (TCs), losses to shipping caused by capsizing are the total losses of ship and cargo,

which are much greater than other kinds of accidents. So, it is resonable to considered the capsizing probability as the risk level in RATC. In recent years, capsizing probability has received attention as an empirical measure to quantify the ship's risk. Shen and Huang (2000) studied the length of time before capsizing and the capsizing probability of ship based on Markov chain theory. Huang et al. (2001) studied the capsizing probability of a ship under the combined action of beam wind and beam sea. In this method, the capsizing probability of every random heeling in an unstable domain is calculated according to the density of heeling extreme value. Thompson (Thompson, 1990; Thompson et al., 1992) used the theory of safe basis erosion to study the ship capsizing probability. Shi et al. (2011) studied the calculation method of ship's movement and capsizing probability in random waves and winds using formula of Gauss-Legendre based on the path integration techniques. Gu (2006) calculated the ship's rolling probability using a new path integration method, which avoided the problem of solving the equations of Fokker-Planck-kolmogorov(FPK). The method of Melnikov is also used to calculate the ship's capsizing probability (Falzarano et al., 1992; Bikdashi et al., 1994; Tang et al., 2004).

3. Mathematical model

The movement of ships sailing on the ocean can be described as the change of ship's position over time. The ship's position ($x(lon, lat)$) and time (t) form the ship's track. The ship's dynamic response to varying environmental conditions can be described by a control vector U and restraint vector M. The ship's heading direction and speed are described by U. The restraint conditions are described by M. The external environment which varies with time is described by E. The dynamic process of a ship can be expressed as:

$$(x,t) = f(x^{'}, t^{'}, E, U, M), \tag{1}$$

in which, $t^{'} = t - \Delta t$, E is the external environment at location $x^{'}$ at time $t^{'}$. The ship is controlled by U during time Δt. The ship will arrive at location x at time t. The economic cost C of the whole voyage can be expressed as:

$$C(x_D, t) = \int_{T_0}^{T} C_{oil}(D, V, Q_a) + C_t, \tag{2}$$

in which, C_{oil} is the fuel consumption per unit time, and related to ship's speed, displacement and the oil price; C_t is ship's profitability per unit time, and related to type of ship and the production plan.

How to improve the economic benefit of a ship based on safety criteria is the concept behind of RATC. An economical and safe RATC based on a dynamic forecasting environment increases ship safety and reduces costs using the wave and wind condition forecasted by numerical models. Costs may be very different given the same risk because of the difference between the ship and cargo's value. To assess the economic benefit of RATC, a benefit-cost ratio for a route is built. A ship's safety can be valued as the safety probability multiplied by the ship's fixed assets including the value of both ship and cargo. Then, the benefit-cost ratio (Ra) of RATC can be expressed as:

$$Ra = \frac{C_{\cos t}}{M_{all} \cdot P_{safe}}, \tag{3}$$

where, $C_{\cos t}$ is the economic cost of the whole voyage, including the time cost and fuel consumption. P_{safe} is the safety probability of ship, considered as the probability that ship will not capsize. M_{all} is the value of the ship and the cargo. Ra is the total cost of ensuring a ship's economic safety in economic cost units when avoiding TCs. Therefore, the smaller the Ra is, the less it takes to ensure a ship's safety when avoiding TCs.

The mathematical model for RATC can be considered as a ship's avoidance of an obstruction that changes shape and position over time. The accident probability of ship must not exceed a acceptable risk level for the operator. Assuming that a ship begins to take an action to avoid TC at time t_0, and will pass through the TC after time T. Then, during time T, the ship's speed and course will change predetermined route. The points at which the ship's speed or course is changed are considered to be waypoints. The avoidance process is regarded as complete when the ship arrives at the next waypoint of the original predetermined route. The whole process of avoiding TCs therefore, can be regarded as an optimal path problem.

Make x_0 as the starting point for ships to avoid TC. $x_n (n = 1, 2, ...)$ are the alternative waypoints of the route; $l_{i,i+1}$ is a segment of the route between two adjacent alternative waypoints; $d_{i,i+1}$ is the distance between any two adjacent alternative waypoints; $p_{i,i+1}$ is

the accidental probability of a ship in a unit time between any two adjacent alternative waypoints; $v_{i,i+1}$ is the average speed-to-ground when ship is sailing on segment $l_{i,i+1}$。Then, the ship's safety probability P_{safe} in the whole process is:

$$P_{safe} = \prod_{i=1}^{n} \left(1 - p_{i-1,i}\right). \tag{4}$$

The economic cost in the whole process is:

$$C_{\cos t} = \sum_{i=1}^{n} \frac{d_{i-1,i}}{v_{i-1,i}} \cdot \left(C_t + C_{oil}\right). \tag{5}$$

The benefit-cost ratio (Ra) of the route is :

$$Ra = \frac{\sum_{i=1}^{n} \frac{d_{i-1,i}}{v_{i-1,i}} \cdot \left(C_t + C_{oil}\right)}{M_{all} \cdot \prod_{i=1}^{n} \left[(1 - p_{i-1,i})\right]}. \tag{6}$$

The acceptable risk level restriction is: $p_{i-1,i} < p_a$.

In which, p_a is the acceptable accident probability of ship capsizing;

i is an alternative waypoint of the route;

n is the amount of the alternative waypoints;

$Q_{oil} \propto D^{2/3} \cdot V \cdot Q_a$ is oil cost in per unit time;

D is displacement of the ship, and Q_a is the oil price.

The most economical route (the minimum economical cost route) based on the safety is $\min\{C_{\cos t}\}$.

4. The RATC algorithm

4.1. Relevant parameters

4.1.1. Ship capsizing probability

Waves are stimulated by the wind during TCs. Wind and waves are strongly correlated. In this paper, the risk factor is simplified using the risk of capsizing in random waves. Some methods are used to calculate the ship's capsizing in waves, i.e. Safe basis erosion (Thompson, 1990; Thompson et al. 1992), Gauss-Legendre (Shi et al., 2011), Melnikov (Bikdashi et al., 1994; Tang et al., 2004).

The differential equation of ship's nonlinear rolling motion in random wave conditions is (Gu,2006b):

$$(I + I_1(\omega))\ddot{\phi} + B_1(\omega)\dot{\phi} + B_2(\omega)\dot{\phi}^3 + \Delta GZ = F_{sea}(\tau), \tag{7}$$

where, I is the rotational moment of inertia around an assumed rolling centre; $I_1(\omega)$ is the added moment of inertia due to the ambient fluid; B_1 and B_2 are linear and cubic damping coefficients respectively; Δ is the ship displacement; GZ is the righting arm of a rolling ship; $F_{sea}(\tau)$ is the external excitation resulting from random beam seas, the over-dots denote differentiation with respect to time τ; ω is the wave angular frequency; ϕ stands for ship's roll angle.

The righting arm is approximated by the following odd cubic polynomial of ϕ,

$$GZ(\phi) = C_1\phi - C_3\phi^3. \tag{8}$$

The excitation moment resulting from the random seas is expressed as:

$$F_{sea} = I_0\alpha_0\omega_0{}^2\frac{\sqrt{2\varpi}}{g}\sum_{n=1}^{N}(n\varpi)^2\sqrt{S}\cos(\omega_n t + \xi_n), \tag{9}$$

in which, $I_0 = I + I_1(\omega)$; α_0 effective wave slope; ω_0 is the natural frequency of ship's roll; ϖ is the interval of wave frequency; S is the excitation intensity of white noise; ξ_n is the random phase angle in $(0, 2\pi)$; g is gravitational acceleration.

When a ship is at sea, the wave encounter angle(χ) is the angle of wave encounter between the heading direction and wave direction as illustrated in Fig. 1. Also shown in Fig. 1 are the ship's breadth, B; the ship's speed, μ; the wave's length, λ. The encounter frequency between wave and ship ω_e is (Tang et al., 2006):

$$\omega_e = \omega - \frac{\omega^2}{g}\mu\cos\chi . \tag{10}$$

The encounter spectrum's relationship with the wave spectrum:

$$S_e(\omega) = \frac{S(\omega)}{1-\left(2\omega/g\right)\mu\cos\chi} . \tag{11}$$

The wave spectrum, with single parameter, as specified by the ITTC (International Towing Tank Conference) (Kaplan, 1966) was used:

$$S(\omega) = \frac{A}{\omega^5}\exp\left\{-\frac{B}{\omega^4}\right\}, \tag{12}$$

where $A = 8.10*10^{-3}g^2$; $B = 3.11/H_{1/3}^2$; $H_{1/3}$ is the SWH.

Therefore, the wave excitation torque for random waves is calculated using the following formula:

$$F_{sea} = I_0\alpha_0\omega_0{}^2\pi\sin\chi\sum_{n=1}^{N}\frac{h_n}{\lambda_n}\cos(\omega_{en}t+\xi_n), \tag{13}$$

in which wave height $h_n = 2\sqrt{2\varpi S\left(n\varpi\right)}$ and wave length $\lambda_n = \frac{2\pi g}{\left(n\varpi\right)^2}$.

After dimensionless treatment, the differential equation for a ship's rolling motion in random seas, in which the white noise is considered, is as following (Gu,2006b),

$$\ddot{x}(t) + \varepsilon\delta_1\dot{x}(t) + \varepsilon\delta_2\dot{x}^3(t) + x(t) - \alpha x^3(t) = \varepsilon f(t), \tag{14}$$

in which, $x=\phi$; $t=\omega_0\tau$; $\varepsilon\delta_1=\frac{B_1(\omega)}{\Delta C_1}\omega_0$; $\varepsilon\delta_2=\frac{B_2(\omega)}{I+I_1(\omega)}\omega_0$; $\alpha=\frac{C_3}{C_1}$; $\Omega=\frac{\omega}{\omega_0}$; $\omega_0=\sqrt{\frac{\Delta C_1}{I+I_1(\omega)}}$; $f(t)=\frac{F_{sea}}{\Delta C_1}$; ε is a very small parameter; the time t is controlled by the natural frequency; $\varepsilon\delta_1$ and $\varepsilon\delta_2$ are linear and nonlinear dimensionless dampers, respectively.

The joint probability density of ϕ and $\dot{\phi}$ can be solved by solving the corresponding Fokker-Planck-kolmogorov (FPK) equation for Equation (14). In this paper the method proposed by Gu (Gu, 2006a) is used to solve the FPK equation. The joint probability is:

$$P_F\left(x_1,x_2\,|\,\phi\right)=a\exp\left\{-\frac{\phi}{D}\left(\frac{x_1^2}{2}-\alpha\frac{x_1^4}{4}\right)\right\}\exp\left\{-\frac{1}{D}\left(\delta_1\frac{x_2^2}{2}+\delta_2\frac{x_2^4}{4}\right)\right\}, \tag{15}$$

in which,

$$\phi=\frac{\int_0^\infty x_2(\delta_1x_2+\delta_2x_2^3)\exp\left\{-\frac{1}{D}\left(\delta_1\frac{x_2^2}{2}+\delta_2\frac{x_2^4}{4}\right)\right\}dx_2}{\int_0^\infty x_2^2\exp\left\{-\frac{1}{D}\left(\delta_1\frac{x_2^2}{2}+\delta_2\frac{x_2^4}{4}\right)\right\}dx_2}, \tag{16}$$

a is normalized parameter which can be calculated using the following formula:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}P_F\left(x_1,x_2\,|\,\phi\right)dx_1dx_2=1, \tag{17}$$

D is the amplitude of excitation.

Then, marginal probability density of ϕ can be calculated based on the joint probability density. Thus, the ship capsizing probability P can be calculated using the following formula:

$$P=\max\left\{\int_{\phi_{v1}}^{+\infty}P\left(x_1,t\right)dx_1,\int_{-\infty}^{\phi_{v2}}P\left(x_1,t\right)dx_1\right\}, \tag{18}$$

in which, ϕ_{v1} is the angle of disappearing positive stability,; ϕ_{v2} is the angle of disappearing negative stability; $P(x_1,t)$ marginal probability density.

4.1.2. Ship speed loss in waves

Ship speed loss in waves has an important effect on the estimate of the ship's location in the design of RATC. If the ship's speed loss is not considered, it will cause error when calculating the ship's position over time. At present, there are many empirical formulas to estimate the ship speed loss due to waves (Aertssen, 1969; James, 1957; Li et al., 2011). In this paper, however, the formula proposed by He and Dong (2009) is used to estimate the ship speed loss in the heavy conditions. The formulas described as follow:

$$\frac{\Delta V}{V}=\frac{12\cdot\frac{B}{T}\cdot C_w}{\left[0.45\cdot\left(\frac{L}{100}\right)^2+0.35\cdot\frac{L}{100}\right]\cdot L}\cdot\%, \tag{19}$$

in which, ΔV is ship's speed loss in waves, unit: kn;

V is ship's design speed, unit: kn;

L is length of two makefasts, unit: m;

B is the ship's breadth, unit: m;

T is the ship's draft, unit: m;

C_w is the index of wave grade, which can be calculated using $C_w = K\cdot T_1\cdot H_{1/3}^2$;

K is the correction factor, which can be calculated using the following formula:

$$K=\begin{cases}-0.05\cdot H_{1/3}+0.9 & \mathrm{L}\le 150\\ 1.3 & 150<\mathrm{L}<200\\ 0.0125\cdot H_{1/3}^2+0.05\cdot H_{1/3}+0.7375 & \mathrm{L}\ge 200\end{cases};$$

$H_{1/3}$ is the SWH, unit: m;

T_1 is the significant wave period, unit: s.

4.1.3. Forecast error of numerical model

Although model forecasting accuracy is increasing, the forecast results from numerical models still have errors caused by inaccurate initial and boundary conditions. If forecasting errors are not considered in designing RATC, the calculation of ship's risk will have significant errors. Dealing appropriately with the forecasting errors in numerical models is important for weather routing. The SWH errors are important when calculating a ship capsizing probability.

In the research of Chu (Chu et al., 2004), the SWH errors from WaveWatchⅢ have a Gaussian-type distribution with a small mean (μ) value of 0.02m, as compared with the T/P altimeter data in the SCS. The RMS error (σ) and correlation coefficient between the modelled (H_m) and observed (H_o) SWH are 0.48 m and 0.90 respectively. In this research, 1330 samples were used for statistical analysis.

Taking the error range into consideration, a ship capsizing probability for the forecast wave height is calculated based on the distribution of the wave model forecast errors. In this paper, the error range was processed using the truncation method. Because the probability of errors bigger than 2 m or less than -2 m is only 0.019, this paper only considers the error range in [-2,2]. The division of the error range ΔE_i is shown in Table 1, and the probability for each error range is Δr_i. According to the forecast value of the wave height and the error range, the ship capsizing probability Δp_i in each error range can be calculated. If the value that the forecast value adds to the error range is less than 0, then the probability in this error range will be 0, because the wave height is greater than or equal to 0. Then, the ship capsizing probability p of ship in the forecast condition C can be calculated using the following formula:

$$p = \sum_{i=1}^{8} \frac{\Delta r_i \cdot \Delta p_i}{\sum_{i=1}^{8} \Delta r_i}. \tag{20}$$

4.1.4. Alternative waypoints

The positioning of the alternative waypoints is important to RATC design. Finding analytic solutions for the minimum economic cost of RATC, however, is intractable since the alternative waypoints can be at any point in the sea. To reduce the computational budget, the alternative waypoints must be artificially restricted by the area of heavy seas and rough weather caused by the TCs. In turn, if the distance between adjacent alternative waypoints is smaller, the route will be more economical but the amount of calculations will increase dramatically. Alternative waypoints can be based on the range of the practical need and therefore, this extent is a second restrictive condition in the proposed model for minimum economical cost for RATC. A sketch of alternative waypoints is shown in Figure 2.

Practically, in the northern (southern) hemisphere, it is much safer to avoid a TC from the left (right) semicircular side. Therefore in the proposed RATC algorithm, only waypoints on the safer, left (right) semicircular side of a TC are considered.

4.2. Algorithm design

The algorithm of the minimum cost route for RATC is as follows (figure 3)：

（1）Assume that the start time of a ship's RATC is t_0; the ship's position is x_0 at t_0, the next waypoint of ship's original planned route is x_{end}. Dividing the segment $x_0 x_{end}$ of the route into n parts of equal length, each point is $x_i (i = 1, 2, ..., n)$, and drawing lines perpendicular to the segment $x_0 x_{end}$ through points $x_i (i = 1, 2, ..., n)$. According to the range of the heavy weather area of a TC, m points are chosen on the left semicircular side of the TC for $x_i (i = 1, 2, ..., n)$. These points $x_{i,j}, (i = 1, 2, ..., n; j = 1, 2, ..., m)$ are the alternative waypoints for a ship's RATC (as shown in Figure 2).

（2）Beginning from point $x_{i-1,j}$（when $i = 1$, $x_{i-1,j} = x_0$）, separately calculate the time and the cost to sail from $x_{i-1,j}$ to $x_{i,k}, (k = 1, 2, ..., m)$. The time $t_{i,j,k}$ and cost $C_{i,j,k}$ sailing from $x_{i-1,j}$ to alternative waypoints can be calculated using the following formula:

$$t_{i,j,k} = \int_{x_{i-1,j}}^{x_{i,k}} \frac{1}{v_{ship}(t, x)} dx \qquad (i = 1, 2, ..., n; j, k = 1, 2, ..., m); \tag{21}$$

$$C_{i,j,k} = \int_{t_{i-1,j}}^{t_{i,k}} C_{oil}(D, V, Q_a) + C_t . \tag{22}$$

Taking the ship speed-loss from waves into account, $v_{ship}(t, x)$ changes over time, external environment, and ship course. The real-time change data for wind, waves, and current can be obtained from the numerical model forecasting results.

(3) Ascertaining if there is an un-navigable area in a segment of the route $x_{i-1,j} x_{i,k}$. According to the environmental conditions of the ship's current position, the capsizing probability can be assessed in relation to the acceptable capsizing probability level. If the capsizing probability is acceptable, go to (4); if it is unacceptable, then $C_{i,j,k} = +\infty$.

(4) Calculating the minimum cost $CC_{i,k} = \min_j(C_{i,j,k})$ and time $tt_{i,k}$ to reach the point $x_{i,k}$. If $CC_{i,k} = +\infty$, $x_{i,k}$ is a unnavigable point that all segments cannot reach, it will not be used to select the next segment.

(5) Making $i = i+1$, and carrying out step two (2) to step five (5) cyclically until $i = n$. All the minimum cost segments that connect to the x_{end} (which is x_{n1}) make the shortest time route to avoid TC, which is $X_0X_1X_2.....X_{end}$. The times that a ship will arrive at each waypoint are $T_0, T_1, T_2, ..., T_n$.

In practice, the fewer the waypoints, the better the route. Fewer waypoints means fewer changes in direction and subsequent speed loss, resulting in more efficient operation. Thus, RATC is optimized to reduce the waypoints with no additional cost. The optimization algorithm is illustrated as follows (figure 4):

(1) The positions of the waypoints, which are the result of the shortest time RATC as discussed in the last section, are assigned to $xx_i, (i = 0,1,...,n)$. The time that a ship arrives at waypoint xx_i is assigned to tt_i and the cost to arrive at this point is CC_i. Make $i = 0$, and continue to the next step;

(2) Starting from xx_i, calculate the cost cc_j occurring when the ship is sailing along the segment xx_ixx_j from the xx_i to $xx_j (j = i+1, i+2, ...)$. At the same time, check for un-navigable areas in segment xx_ixx_j in relation to changing wind and wave conditions. Let $z = j-1$ if there is an un-navigable area.

(3) Let $dc = cc_z - cc_i$, and $DC = CC_z - CC_i$. If $dc \le DC$, then $i = z$. Assign xx_i and CC_i to set x_ok and c_ok respectively. Continue to the next step. If $dc \ge DC$, let $z = z-1$ and do step three (3) cyclically.

(4) Let $i = z$, do step two (2) to step four (4) cyclically until $z = n$. x_ok are the waypoints, and c_ok is the cost for the ship as it arrives at waypoints in the optimized shortest RATC.

5. Case study and results

The TC Nockten was chosen to test the algorithm. Nockten's track for every six hours is shown in Figure 5. Nockten made landfall at Wenchang, Hainan province in China. The centre of the greatest wind speed is up to 25 m/s when landing. The direct economic loss caused by Nockten is estimated at about 0.6 billion dollars and two people were killed. Nockten also exerted serious influence on ships at sea.

Assume that a ship was in 19N, 111.5E at 0000 UTC, 28 July 2011, and that the ship took action to avoid the TC as soon as it received the TC warning. The next waypoint of the original route was at 20.5N,120E. In this study, this scenario was used to design the RATC. The ship's value was 23.8 million dollars while the value of the cargo was 34.9 million dollars. The price of oil was $871.00 per ton. The ship's profitability was 11.1 thousand dollars per day. The fuel consumption was 24 tons per day, at a speed of 10 knots (kn). It is assumed that the change of the ship's displacement was ignored in the RATC and that the ship's speed through water was unchanged.

5.1. Numerical TC simulation

The WRF model was used to simulate the TC. In the model, the 1°×1° NCEP (National Center for Environmental Prediction) data for 10:00 on 28 July was used as the initial data for the simulation, and six hour time interval daily NCEP data was used as boundary data. The model simulation area was 99°E-130°E，0°-30°N. The resolution was 0.1°×0.1°, time step was 300s. The wind field as forecasted by WRF was used to drive the WaveWatch-Ⅲ . The resolution of the wave model was 0.1°×0.1°, time step was 900s. The resolution of wave direction was 15°. The results of the two models are shown in Figure 6.

5.2. The ship's capsizing probability

To calculate the ship's capsizing probability, the integration time was set to 0-300s; time step was set to 0.0125s; N was set to 180; the upper limit of the wave power spectrum was set to 4.5rad/s; the frequency interval ϖ was set to 0.025rad/s; α_0 was set to 0.729. The change in the ship's capsizing probability with changes in the ship's heading and wave direction are shown in Figures 7a. Given the same wave height, the ship's capsizing probability reaches a maximum when the wave direction is close perpendicular to the ship's heading. The ship's capsizing probability is higher with increasing wave heights (Figure 7b). The change in a

ship's capsizing probability with the wave height and the angle between the heading and wave direction is show in Figure 7c.

The ship capsizing probability under different wave and heading conditions at 1000 28th is shown in Figure 8. The Figure shows that ship capsizing probability is very different when the angle between ship's heading direction and wave direction is different. So, the RATC must consider the angle between ship's heading direction and wave direction. The ship's capsizing probability is small with stern waves or when the vessel is sailing head to sea.

5.3. Economy and safety route design

The ship's capsizing probability at different heading directions and times were calculated based on the forecast results from the numerical models. The alternative waypoints are shown in Figure 2, where the interval in the perpendicular direction of the original route is 0.35°. The ship speed through water kept constant. The routes of different acceptable capsizing probability and ship speed are listed in Table 3. To compare the model's superiority to the sector diagram typhoon avoidance method, the same experiment is done using sector diagram typhoon avoidance method. In figure 9, A, B, and C separately represent the location of the tropical cyclone center at 00:00, 06:00, and 12:00 on 28 July (UTC). H_1, H_2 and H_3 represent the ship's location at 00:00, 06:00, 12:00 on 28 July (UTC). The result of sector diagram typhoon avoidance method is Exp6. The ship's experimental RATCs are shown in Figure 10a.

Exp1~Exp3 (the ship has the same acceptable capsizing probability but different ship speed) are taken as a group which are used to test the ship speed's effect on the result. Although the speed in the Exp3 is higher (1kn) than Exp2, but the shipping time is only less 0.1 hour than Exp2. This is because that when the ship speed is higher the environment which it countered is different, it has to choose the route according to the changing environment. For Exp3 (Figure 10a), the ship have to travel long distance to avoid the higher risk area (capsizing probability). It shows that higher speed does not mean less shipping time as expected. In the acceptable capsizing probability, the safety probability of the RATC is also different with ship speed. The middle speed (16kn) has the highest benefit-cost ratio. The benefit-cost ratio is no direct relationship with the ship speed. Exp3~Exp5 are taken as a group to test the effect of acceptable capsizing probability on the result. In this group, higher risk (capsizing probability) can reduce the shipping time. But it does not mean higher benefit-cost ratio. The benefit-cost ratio of Exp3 is the highest in this group, the risk is the middle one in this group. Comparing with the Sector diagram typhoon avoidance method (Exp6), the model can reduce the ship's risk when avoid TC although the cost. When the ship

speed is same, the benefit-cost ratio of RATC is higher than Sector diagram typhoon avoidance method (see Exp3-Exp7).

In Figure 10b, the ship's speed is 17kn and the acceptable capsizing probability of ship is 7×10^{-4}. The blue line is the RATC before optimization while the red line is the route after optimization. Figure 10b shows that the optimal route can reduce more waypoints. Figure 11 indicates the ship's position at different times, colours as shown in the capsizing probability scale bar at the bottom of the figure, represent the ship's capsizing probability at several heading angles at different times. The RATC in different time can avoid the area of the high risk with changing environment.

Experimental results show that: the higher the acceptable capsizing probability level, the lower the cost of the RATC. Meanwhile, the higher risk to the ship must be considered. The higher ship speed does not mean the low time cost as expect. However, lower costs do not mean that the route's benefit-cost ratio is higher. The case study shows that the cost of a ship's RATC is related not only to the ship's speed, but also to the ship's the acceptable risk level. In same environment condition, the ship's risk is related with ship's performance. When ship's accept risk (acceptable capsizing probability) is chosen, the suitable ship speed can be make according to the benefit-cost ratio of the RATC. Compared to the Sector diagram typhoon avoidance method, the routes provided by this model are more safe and economical.

6. Conclusions

TC is the serious threat for ship sailing in ocean. How to choose the route to avoid TC safety and economic is an essential subject for the sailor and ship company. The high accuracy forecasted environment data from numerical models is possible to guide RATC. The traditional methods to find RATC have some limits. For example, they don't quantify the risk according the ship's performance, but according the sailor's experience to estimate the ship's risk. In this paper, a safe-economical RATC was designed based on the dynamic forecast environment. In this proposed route design method, the limitations of the traditional methods to find RATC are avoided or reduced. The ship's risk under heavy seas is quantified using the ship's capsizing probability where the ship's capability to resist wind and waves is considered. Although the accuracy of forecast environment data has improved compared with past, the forecast error is still needed to consider. In the model, when calculated the risk the effect of the forecast error are considered according to the error distribution. Ship speed loss is also taken into the model, which has a big effect on calculating ship's position with time.

An acceptable risk level should be set according to the ship's characteristics and the company's risk tolerance. Based on the acceptable risk level and the benefit-cost ratio of

different ship speed, the best speed to avoid TC can be figure out according to their benefit-cost ratio. According to the case study, comparing the traditional method (Sector diagram typhoon avoidance method), this model not only ensures a ship's safety but also reduces shipping costs. Using the model, companies and sailors can design RATC according to the ship's performance. It can increase the company's benefit within the ship's acceptable risk. There are some aspects needed to be considered in the future: (1) the joint effect of wind and wave will be included when calculating the ship's capsizing probability. In this paper, we only consider the wave's effect on ship's capsizing probability. The joint effect of wind and wave will be more accurate when calculating the capsizing probability. (2) the rapid growth of computing costs when increasing the spatial resolution of alternative waypoints will be addressed. When the alternative waypoints become more, there will be more calculation for the ship capsizing probability, the waypoint choice and the shipping time comparison. It is necessary to design an intelligence algorithm to solve the problem. (3) the comfort level of crew.

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Table1 The forecast error range division

forecast error range(in meters)	[-2,-1.5]	[-1.5,-1]	[-1,-0.5]	[-0.5,0]	[0,0.5]	[0.5,1]	[1,1.5]	[1.5,2]
Probability in this error range	7.59e-4	1.6e-2	0.12	0.34	0.36	0.14	0.02	0.001

Table 2 Parameter values of the ship

L	171.7m	I_0	$1.070*10^7 kg.m^2$
B	16.8m	C_1	0.871m
Δ	8000t	C_3	0.013m
B_1	$1.070*10^7 kg.m^2/s$	ϕ_{v1}	-1.39rad
B_2	$1.070*10^7 kg.m^2$	ϕ_{v2}	1.39rad
ω_0	0.807rad/s	T	8m

Table 3 Experimental results

No.	Ship speed (kn)	Acceptable capsizing probability Pa	Maximum wind in the sailing (m/s)	Maximum wave in the sailing (m)	Shipping time (hour)	Safety probability	Cost (thousand dollar)	Logarithm of Benefit-cost ratio $\lg(Ra)$
Exp1	15	0.001	18.1	5.2	46.6	0.9991	81.37	-2.858
Exp2	16	0.001	17.8	5.3	41.6	0.9992	76.60	-2.884
Exp3	17	0.001	20.1	4.4	41.5	0.9991	79.71	-2.867
Exp4	17	0.002	20.1	4.4	40.7	0.9980	78.17	-2.875
Exp5	17	0.0007	20.1	4.4	41.5	0.9993	79.71	-2.867
Exp6	17	/	20.7	4.2	40.9	0.9959	78.56	-2.866

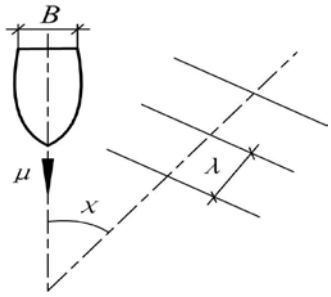

Figure 1 The wave encounter angle

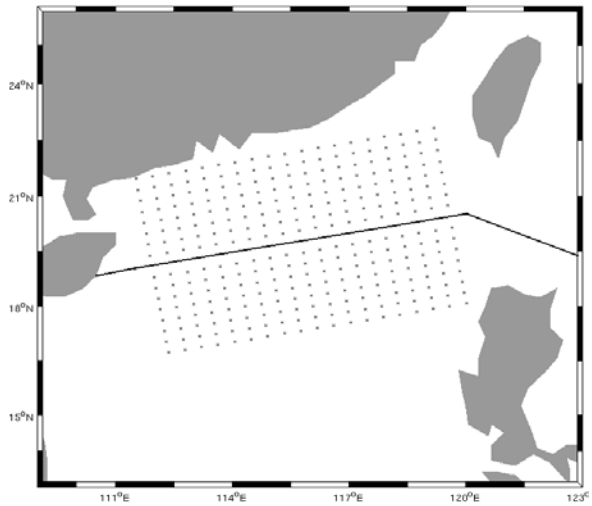

Figure 2 Sketch of alternative waypoints

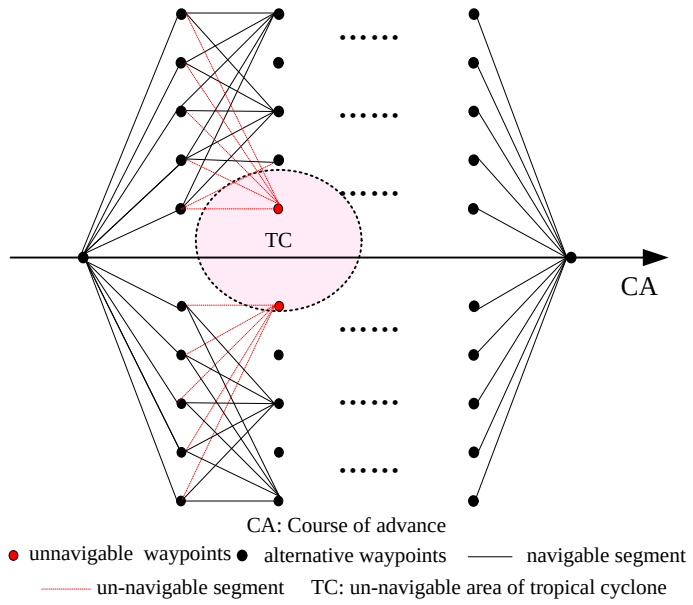

Figure 3 A route designed to avoid TC

(The unnavigable waypoint is the point that all segments cannot reach)

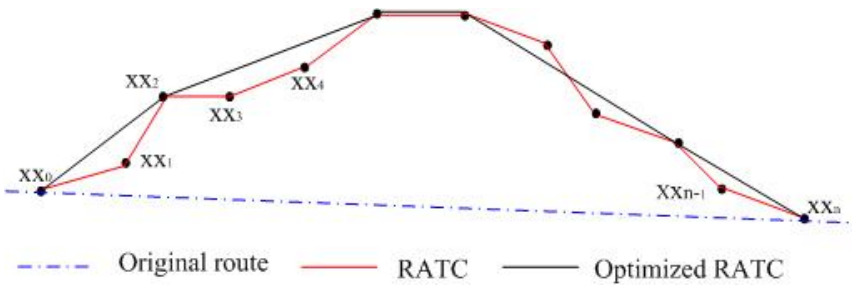

Figure 4 Optimized RATC

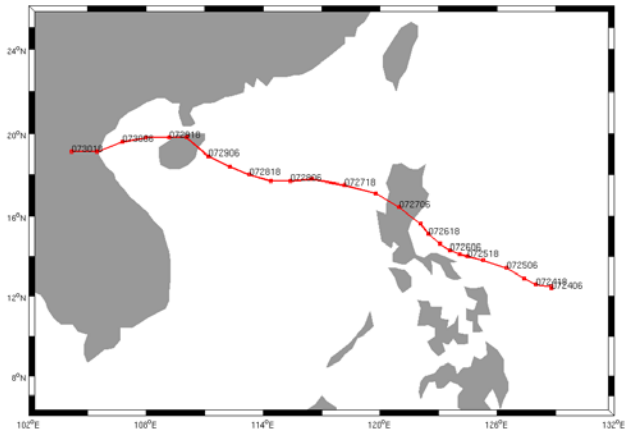

Figure 5 The track of TC Nockten every six hours

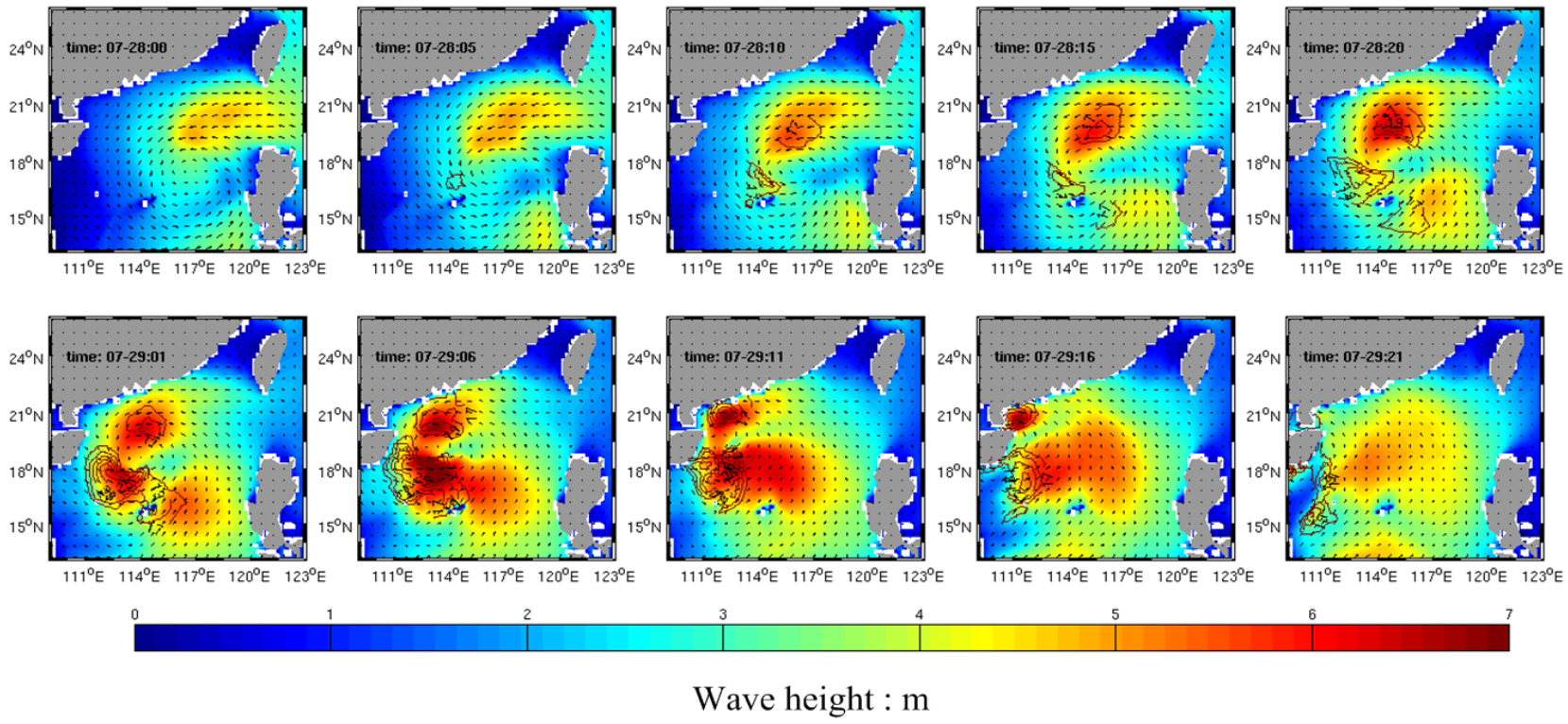

Figure 6 The forecast wind and wave conditions

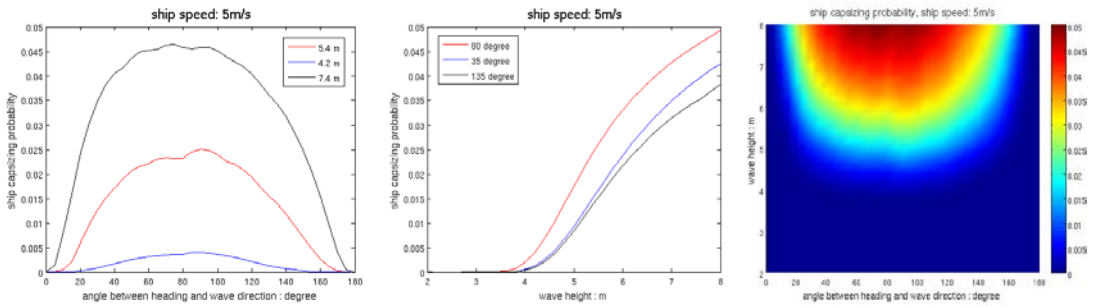

Figure 7 The change of ship's capsizing probability

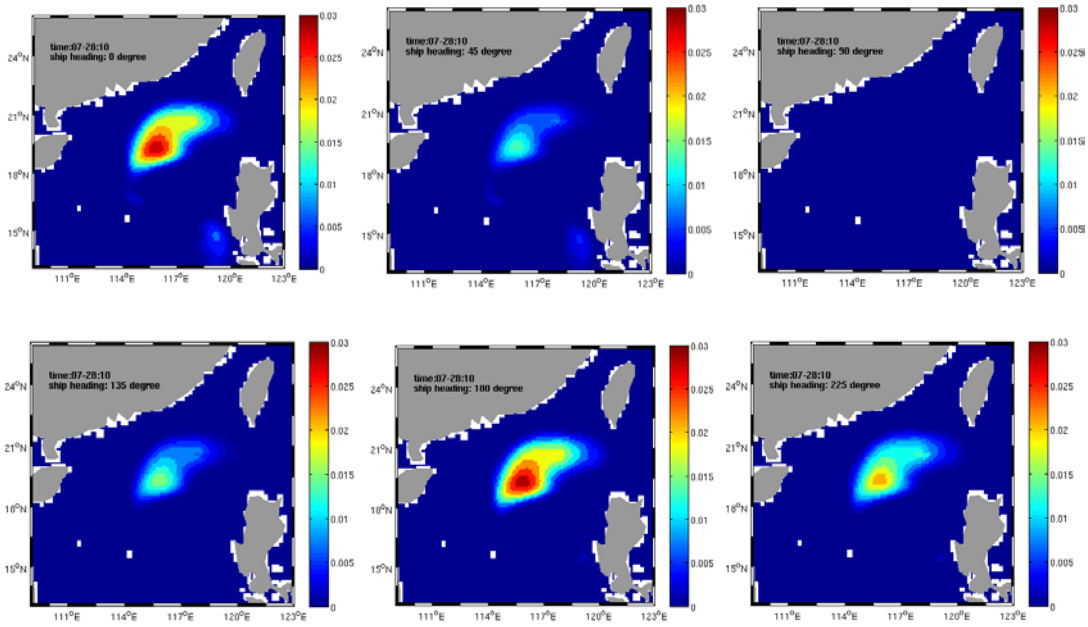

Figure 8 The ship's capsizing probability at different ship headings on 28 July 2011, 10:00 (UTC)

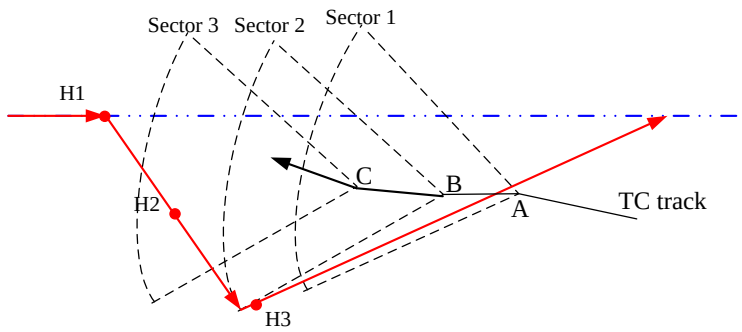

Figure 9 Sector diagram typhoon avoidance method

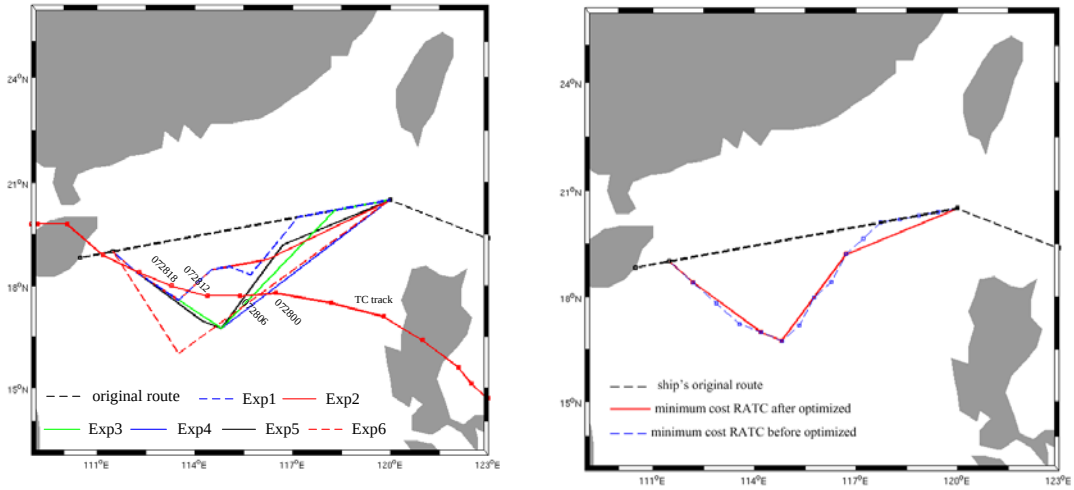

Figure 10 (a) Experimental routes (b) The route before and after the optimization experiment Exp5

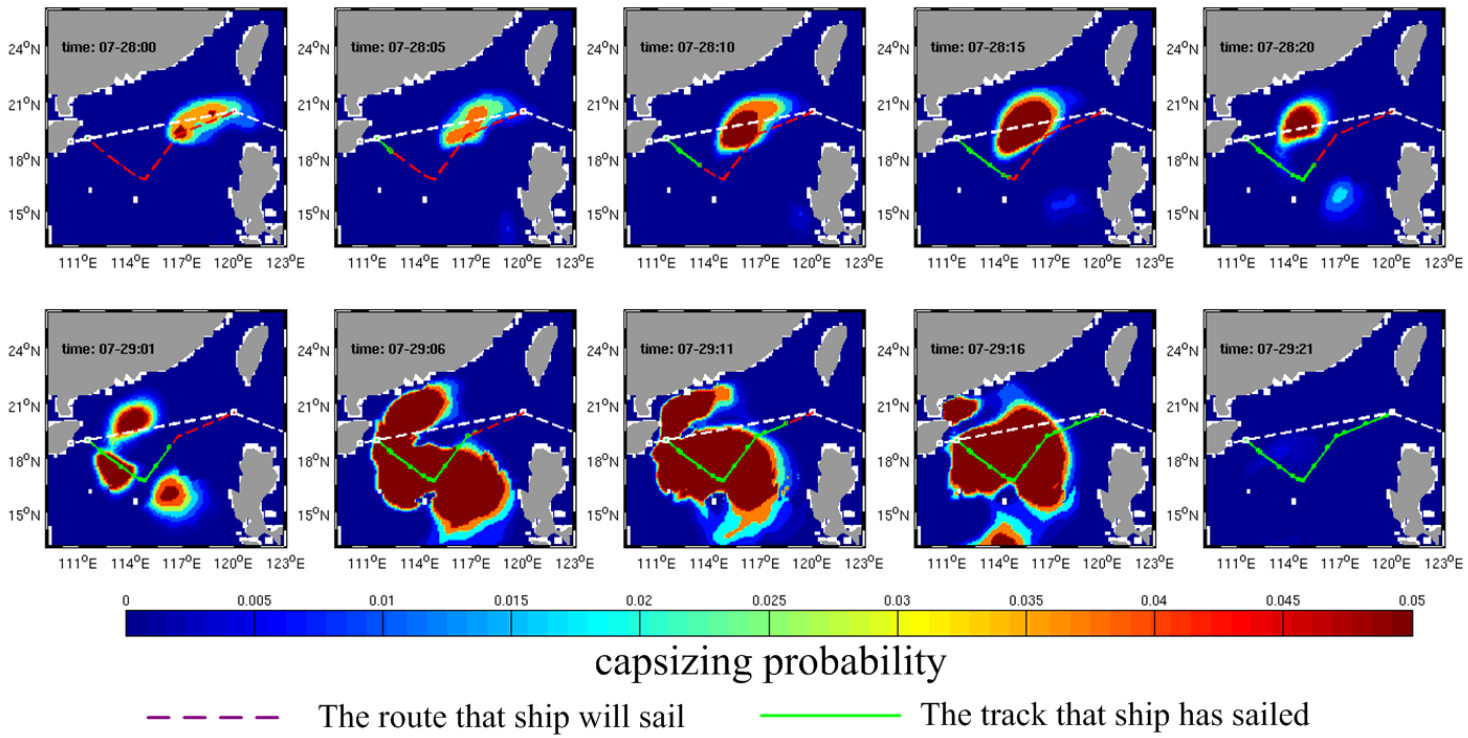

Figure 11 Ship position and the capsizing probability at different times