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# A wavefront orientation method for precise numerical determination of tsunami travel time

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895

## Abstract

We present a highly accurate and computationally efficient method (herein, the "wavefront orientation method") for determining the travel time of oceanic tsunamis. Based on Huygens principle, the method uses an eight-point grid-point pattern and the most recent information on the orientation of the advancing wave front to determine the time

for a tsunami to travel to a specific oceanic location. The method is shown to provide improved accuracy and reduced anisotropy compared with the conventional multiple grid-point method presently in widespread use.

## 1 Introduction

- Determining tsunami travel time is one of the fundamental roles of tsunami warning centers and other agencies tasked with estimating the possible impact of approaching tsunami waves on different coastal regions. In contrast to most other natural hazards, tsunami travel-time calculations can be performed well in advance. Because the travel time is the same when the source and the target are inverted, the inverse travel time
- <sup>15</sup> for populated coastal sites have been computed and the corresponding databases of inverse travel-time maps created. When there is a tsunami event, the travel time to a specific coastal site is available immediately once information has been received about the initial parameters of the underwater earthquake and its location. Any initial travel time estimation will be preliminary (it is not based on the actual extent of the
- <sup>20</sup> tsunami source) and needs to be refined as new information regarding the tsunami source becomes available.

In addition to the need for accurate real-time travel-time estimates for tsunami alerts and warnings, highly reliable estimates of tsunami travel time are required for optimizing the location of offshore tsunami warning stations used for refining information on

the parameters of approaching tsunami waves (Poplavski et al., 1988). These stations require accurate advanced knowledge of the difference in wave travel time between

Discussion Paper | Discussion Paper

Discussion Paper

Discussion Paper

specific coastal locations and the warning station (Titov et al., 2005). The greater the distance between the tsunami station and the coast, the greater the advanced forecast time for distant remotely generated tsunamis but the shorter the forecast time for locally generated tsunamis. The optimal warning system design should, therefore, be based

- on extensive determination of wave propagation times which, in turn, requires an effi-5 cient and accurate method for calculating the tsunami travel time. Accurate estimation of observed wave arrival times is also needed for research delineating the tsunami source region. Numerical modeling of the travel times for near-source locations can also provide important information concerning the size and shape of the source re-
- gion (e.g. Abe, 1973; Fine et al., 2005; Hayashi et al., 2012). Lastly, any discrepancy between the computed and observed tsunami travel times may indicate that the geometry of the source region is incorrect or that there maybe physical effects, such as wave dispersion, nonlinearity, and coupling with elastic earthquake modes, which should be taken into account. These affects often require better wave travel time accuracy than 15 for tsunami warning purposes.
  - There are presently two primary approaches for calculating tsunami travel time from gridded bathymetry: (1) by kinematic wavefront propagation calculations based on Huygens principle (Shokin et al., 1987); and (2) by solving the dynamical equations of motion, typically using finite-difference method (cf. Kowalik et al., 2005). The first method is
- the one most commonly used in modern tsunami travel time calculations and is favored by commercial software such as GEOWAVE (GEOWARE, 2013). Following Huygens principle, each of the points along a wave front is a source for the tsunami waves. These points serve as start locations for travel time computations to the next N neighbouring points. In GEOWAVE, N can have the values 8, 16, 32, 48 or 64, and the tsunami travel
- time (TTT) between points is computed using the shallow water propagating speed for-25 mula,  $c = \sqrt{gh}$ , where g is the gravitational acceleration and h is the water depth. Water depths are obtained from the gridded bathymetry provided by ETOPO (Amante and Eakins, 2009). If the arrival time to the nearest point is not determined, or if the previously computed arrival time is greater than the currently computed arrival time,

the arrival time at the latter point is replaced with the newer value. At the end of the computational step, a new source point is specified as that frontal point that had the minimum travel (and arrival) times. This frontal point is converted to a permanent point to be used as source point for the next time step.

- A critical step in the calculation is defining the pattern of neighbouring points. The larger the pattern, the greater the isotropy of the wave field, and the better the accuracy of the timing calculation for open ocean waves. On the other hand, a larger pattern also creates a broader region of uncertainty along the frontal zone which, in addition to the computational overhead, creates problems in zones where the water depth is changing
- quickly. Moreover, a larger grid pattern can cause the tsunami arrival estimations along 10 certain angles to "jump" over shallow water areas or over islands, leading to the need to integrate the travel time over lines connecting the two points.

The wavefront orientation method proposed in this study uses a less extensive grid pattern by taking advantage of the most recently gained information on the tsunami

propagation in the neighbouring grid points. In particular, the method uses knowledge about the local direction of wave front propagation obtained from previous calculations and, as a result, yields a more accurate travel time calculation without the need to apply a large pattern of grid points.

#### 2 Methodology

#### 2.1 The conventional method

The conventional method calculates tsunami travel time for each time step by dividing the spatial grid into three categories: (A) grid points for which the travel time has already been computed; (B) intermediate, or frontal, grid points where the travel time is to be evaluated; and (C) grid points that have yet to be reached by the advancing waves. At

each time step, GEOWAVE starts the calculations from specific grid points in category A and then calculates arrival times for N neighbouring points in categories B and C

- For any given grid pattern, there are errors in the conventional method due to the fact that not all propagation directions from the different source regions to the new grid point are covered by the pattern. Moreover, because of the structure of the rectangle pattern, the directional coverage is not uniformly distributed. As indicated by Fig. 1, the biggest gaps are in the directions along the grid axes, corresponding to those directions in-line
- with the grid points (i.e. between the direction from centre O to the point B1 and to the point B2, or between directions from point O to the point B24 and to the point B25, and so on); consequently, the main arrival errors are in these directional sectors. Note that we are restricting our analysis to the case for uniformly propagating wave velocity, where the relative errors in travel time are equivalent to the relative pathlength errors.
- Directional errors for the conventional method can be estimated analytically. Each pattern has a fixed number (or list) of directions of propagation,  $\alpha_i$  (i = 1, 2, ...), corresponding to the grid pattern number N. If the directional angle  $\beta$  from the source to a given point is not in the list, the fastest way to reach a given point in the model is along one of the listed directions,  $\alpha_k$ , and then in the adjacent direction  $\alpha_{k+1}$  such that

<sup>20</sup> 
$$\alpha_k < \beta < \alpha_{k+1}$$
.

The resulting accumulated distance for the two-direction pass will be greater than the distance between the source and the given point. The relative increase in path length,  $\delta$  (corresponding to an increase in the tsunami travel time), is

$$^{25} \quad \delta = \frac{\sin(\beta - \alpha_k) + \sin(\alpha_{k+1} - \beta)}{\sin(\alpha_{k+1} - \alpha_k)}.$$

899

The relative change in pathlength,  $\delta$ , is a maximum when the direction of propagation,  $\beta$ , is equal to the mean of the bounding list directions  $\alpha_k$  and  $\alpha_{k+1}$ 

$$\beta_{\max} = (\alpha_k + \alpha_{k+1})/2,$$

5 whereby

δ

$$\max = \frac{1}{\cos(\alpha_{k+1} - \alpha_k)}.$$

The angle  $\beta_{\text{max}}$  and corresponding relative errors err =  $\delta$  – 1 are listed in Table 1. Results for the conventional method are listed as P8, P16, P32, P48 and P64 according to the number of points *N* used in the travel time computations.

Errors for the conventional method are always positive or zero, and only zero in those directions lying along the direct interconnection between grid points. Based on Table 1, the directional errors are typically small, and decrease with an increase in the number of points used in the computational grid pattern. However, because the relative error

15 remains uniform with distance, the absolute error can become significant over long distances, such as in the case of trans-Pacific tsunami propagation.

## 2.2 The wavefront orientation method

We propose a new methodology for calculating tsunami travel time which, in addition to a small spatial grid pattern, makes use of the most recently derived information

- <sup>20</sup> regarding the tsunami travel time at the point closest to the current source position. Let O be the current source point and let A5 be the nearest point for which the travel time has been determined (Fig. 2). Because the time it has taken the tsunami to reach the current source position is, by definition, a maximum, the difference in travel (or arrival) time  $\Delta t_{OA5} = t$ (Point O) – t(Point A5) > 0. Let  $c^{-1}$  be the wave slowness (inverse of the wave speed, c). We assume that the tsunami wave front is a straight line in the vicinity
- wave speed, c). We assume that the tsunami wave front is a straight line in the vicinity of the source point, and that the tsunami arrival time, t, in the triangular region O-A5-B7

(1)

(2)

**Discussion** Paper

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(4)

in Fig. 2 is a linear function of the local spatial coordinates x and y, such that

$$t(x,y) = \frac{x\cos(\phi) + y\sin(\phi)}{c} + t(0,0),$$
(5)

where the orientation angle,  $\phi$ , of the tsunami wave front can be found using known values of the time difference,  $\Delta t_{OA5}$ , corresponding to the arrival time between points O and A5. Specifically,

$$\cos(\phi) = -c \frac{\Delta t_{\text{OA5}}}{\Delta x},$$

where  $\Delta x$  is the incremental grid step in the x direction. The travel time to point B7 will be

$$t(\text{Point B7}) = t(\text{Point O}) + \Delta y \sqrt{\left(\frac{1}{c}\right)^2 - \left(\frac{\Delta t_{\text{OA5}}}{\Delta x}\right)^2},$$
 (7)

where  $\Delta y$  is the grid step along the y direction.

- Equation (7) allows for more accurate computation of the travel time to point B7 than the conventional method. However, because it is based on previously computed travel times at two points, our method requires some initialization, which can be provided by the conventional method using the simple 8-point algorithm. Unlike the conventional method, this new approach has no fixed directional errors. As a consequence, relative errors decrease with distance from the start location (origin) of the tsunami event. This
- <sup>20</sup> provides an improvement over the conventional method whose level of error depends on the initialization pattern.

901

#### 3 Method comparison

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#### 3.1 Constant depth ocean on a spherical geographical grid

One of the primary concerns in tsunami research is determining the travel times for trans-oceanic (mostly trans-Pacific) tsunamis. It is, therefore, of interest to determine

- <sup>5</sup> how our newly proposed method performs relative the conventional method for a spherical, constant-arc step-grid for the Pacific Ocean. We first examine the method's performance for an ocean of constant depth and then compare the results to those for an exact analytical solution. Table 3 shows the absolute errors in tsunami travel time obtained for an ocean of 4000 m depth for both methods.
- According to Table 3, the maximum timing error decreases with grid pattern number, but remains measurable even for the conventional P64 algorithm. The errors are significant for the P32 and, especially, the P16 algorithms. In addition, the conventional method has a positive bias, which needs to be corrected. The GEOWARE software decreases this bias by introducing a "correction coefficient". In contrast, the proposed
- <sup>15</sup> wavefront orientation method is almost free from errors; the maximum differences between the numerical results and the theoretical (analytical solution) values do not exceed 12 s over the entire propagation distance of 15 000 km.

Figure 3 shows the distribution of errors for the different methods. As for the plane wave case discussed in Sect. 2, the accumulated timing errors for the conventional method generally increase with distance, although the distribution of errors differs from that for the plane wave case. In particular, the errors concentrate near both sides of

- the latitude meridians passing through the source region but are almost absent near the parallels of longitude passing through the source region. This is because, for high latitudes, the wave propagation beams are not straight lines in the spherical coordi-
- nate system (except in the meridional direction). Specifically, the beams can't be along parallels but must be along great circles. In the near-meridional direction, the relative error is similar to that for the plane wave case; i.e. about 0.5%, 1.3% and 2.7% for the conventional grid cases P64, P32 and P16. For the new wavefront model the error

(6)

Discussion Paper

Discussion Paper

Discussion Paper

Discussion Paper

distribution has little directional bias mainly because it is an almost error-free calculation to begin with.

## 3.2 Pacific Ocean with realistic seafloor bathymetry

For estimations of tsunami travel time under realistic ocean conditions, accurate measurements of water depth are of major importance. In addition to its direct affect on the actual travel time, changes in water depth can affect the direction error as the wave beams undergo convergence and divergence. As shown by Satake (1988), these effects completely change the beam pattern of tsunami energy flux in the Pacific Ocean compared to that for an ocean of constant depth. The difficulty with estimating the

- reliability of the tsunami travel time calculation over a variable depth ocean is that it cannot be compared to an analytical solution. Moreover, because of measurement uncertainties, the observed travel times to tsunami recording sites such as tide gauges and bottom pressure recorders are not sufficiently accurate to judge the numerically derived results. We are, therefore, limited to comparing differences between the differ-
- ent numerical methods, taking into account the hierarchy of the conventional method, denoted as models P8 to P64, for which the accuracy increases with increasing pattern number.

We have run computations for a realistic ocean using the identical source position as for the constant depth ocean (i.e.  $175^{\circ}$  W,  $50^{\circ}$  N) and covering the same spatial

<sup>20</sup> domain (lower left corner: 95° E, -72° S and upper right corner: 80° W, 65° N). We have also used the same ETOPO-1 dataset and same GEOWARE software for both the conventional and wavefront method. To avoid complications arising from different interpolation schemes in coastal areas, we have restricted our comparisons to open ocean points only, with depths greater than 3000 m. Results of these comparisons are presented in Table 4 and Fig. 4.

Because the conventional method always overestimates the tsunami travel time regardless of the number of grid points used, the travel time errors using conventional model P16 versus using model P64 (written P16–P64) or model P32 versus P64

903

(P32–P64) are on the low side relative to the actual analytically derived timing errors. For this reason, the errors for conventional model P16 versus our wavefront orientation model F8 (P16–F8) are greater than those for P16–P64. Similar travel time errors apply to P32–F8 versus P32–P64. As a consequence, the errors presented in the first 2 rows
 of Table 4 closely correspond to those presented in the first 2 rows of Table 3.

- As shown by the bottom 3 rows of Table 4, The difference in travel time P16–F8 is close to the difference P16–P64, while the differences for P32 versus F8 are close to the P32 versus P64 differences. The last row in Table 4 shows differences between the best conventional method results and the wavefront orientation method results. These
- values are clearly quite low, with maximum and minimum differences only 2.5 min and -1.1 min, respectively. On the other hand, the differences P16–F8, P32–F8 and P64– F8 are consistent with the first 3 rows of Table 3 (which compares P16, P32 and P64 with the exact solutions for a constant depth ocean). This indicates that F8 (which combines the P8 algorithm with the wavefront orientation algorithm) outperforms all of
   the conventional runs, including P64, even for the variable depth case.
  - Figure 4 shows the difference distributions presented in the same order as in Table 4. Generally, the distributions for the cases P16–F8, P32–F8 and P64–F8 (Fig. 4a1, b1 and c1) are similar to the error distributions shown in Fig. 3 (a, b, and c, which represent the differences between the conventional method and the exact solution for
- <sup>20</sup> the constant depth ocean case). This again confirms that the new method provides more accurate time of arrival values that the conventional method.

#### 4 Discussion and conclusion

The previous results demonstrate that our proposed "wavefront orientation method" provides a more effective and accurate methodology for calculating the travel times for trans-oceanic tsunamis compared to the conventional method. Moreover, our method works especially well for large bathymetric grid regions; the larger the gridded array,

the more advantageous the method.

It is important to note that the accuracy of our newly proposed method should not be confused with the general accuracy of tsunami travel time calculations. Actual tsunami travel times depend on many factors that are independent of the particular algorithm being used (Wessel, 2009). Specifically, actual travel time accuracy depends on the ac-

- <sup>5</sup> curacy of the bathymetric dataset. In the open ocean, present gridded global datasets are sufficient for most practical applications. However, this is not the case for coastal areas. Moreover, local, high-resolution bathymetric data are not fully incorporated in the ETOPO datasets provided with the GEOWARE software. This can lead to significant errors in the travel time estimates. In addition, global datasets do not resolve narrow
- passes in coastal areas, requiring the use of local nested bathymetric data. The limitations of the bathymetric data are compounded by the fact that the tsunami generation region is typically poorly defined. In a scientific twist, observed tsunami travel times are often needed to better define the source region that generated the waves in the first place.
- There are other physical factors which can cause the actual speeds of tsunami waves to depart from wave speed estimates obtained from simple linear shallow water theory. One of the more important factors is frequency dispersion related to non-hydrostatic effects. Non-hydrostatic effects are especially important during far field wave propagation than during near-field tsunami wave propagation, or when the tsunami source area
- is relatively small (shorter wavenumber waves) and/or located in relatively deep water (González and Kulikov, 1993). Although non-hydrostatic effects modify tsunami wave propagation in a depth-variable ocean, the travel times for non-hydrostatic waves can be computed in the same way as for shallow water theory by examining each wave frequency separately. This is because, for a given frequency, the arrival time is determined
- by the group velocity of the dispersive waves, which depends on the depth only. In shallow water regions, frequency dispersion becomes less important. However, nonlinear effects arising from wave interaction with the tsunami wave field, and from interaction between the tsunami waves and tidal currents, can modify the travel times. In estuarine areas, tidal motions and river flow may also alter the travel times. As shown

by Watanabe (2012), it is also possible to have interactions between ocean gravity waves and elastic oscillations in Earth's mantle and between acoustic waves in the ocean. Both factors can generate noticeable changes to tsunami travel times.

Regardless of the factors affecting tsunami wave propagation, it is clear that accurate estimation of tsunami travel time is of considerable importance. Because the Huygens principle does not depend on wave speed formulations, the above factors (excluding perhaps non-linear effects) can be included in our travel time algorithm. Thus, in addition to providing improved computational efficiency and better accuracy compared to the conventional methodology, our proposed method can be used to enhance general tsunami research.

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10

907

**Table 1.** Directional error parameters for the conventional method (denoted by the letter "P") applied to a plane square grid. RMS is the root mean square of the error and STD is the standard deviation of the errors. Column 2 gives the angle yielding the maximum error.

Method	Angle $\beta_{\max}$ , degrees	Maximum error (%)	Mean error (%)	RMS (%)	STD (%)
P8	22.5000	8.239	5.477	6.006	2.4624
P16	13.2825	2.749	1.438	1.6581	0.8260
P32	9.2175	1.308	0.490	0.6442	0.4184
P48	7.0181	0.755	0.245	0.3325	0.2250
P64	5.6550	0.489	0.149	0.2054	0.1417

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**Table 2.** Relative errors for the proposed wavefront orientation method and a plane square grid. Distance from the source location at Point O (0,0) is measured in terms of the specific point number on the grid. The larger the grid number, the greater the distance from the source region. RMS is the root mean square of the error and STD is the standard deviation of the errors.

Average distance from the source, points	Maximum error (%)	Mean error (%)	RMS (%)	STD (%)
96–104	0.303	0.188	0.210	0.093
496-504 996-1004	0.091	0.056	0.062	0.027
4996–5004	0.013	0.018	0.010	0.003

909

**Table 3.** Errors in tsunami travel time for a constant (4000 m) depth ocean with bathymetry gridded at 1-arc minute steps for both the conventional and proposed methods. Values are compared to exact values obtained from an analytical solution. Results for the conventional method are denoted with a "P"; those for the waveform method with an "F". RMS is the root mean square of the error and STD is the standard deviation of the errors.

Method	Minimum error (min)	Maximum error (min)	Mean error (min)	RMS (min)	STD (min)
P16	0	17.0	7.1	8.2	4.0
P32	0	8.5	3.1	3.7	1.9
P64	0	4.2	1.6	1.8	0.8
F8	-0.1	0.2	0.1	0.1	0.03

**Table 4.** Statistical parameters for differences in computed trans-Pacific tsunami travel times using the ETOPO-1 dataset for the conventional and the proposed wavefront orientation methods. F8 denotes runs using the new waveform orientation method based on the 8-point GEOWAVE algorithm. RMS is the root mean square of the error and STD is the standard deviation of the errors.

Methods	Minimum difference in time (min)	Maximum difference in time (min)	Mean difference (min)	RMS (min)	STD (min)
P16–P64	0	19.4	4.9	5.8	3.0
P32–P64	0	6.3	1.4	1.7	1.0
P16–F8	-0.1	20.0	5.6	6.5	3.2
P32–F8	-0.2	7.8	2.1	2.4	1.3
P64–F8	-1.1	2.5	0.7	0.9	0.5

911



**Fig. 1.** Schematic showing one time step in the calculation of tsunami travel time for a 32-point pattern for the conventional method. Grid points are divided into three categories: (A) points for which the travel time has been calculated (dark grey area); (B) points in the frontal zone (light grey area) for which the time is being calculated; and (C) points where no calculation has yet been made (unfilled area).



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913





Fig. 3. Maps of tsunami travel time errors (min) for a uniform depth (4000 m) ocean using: (a-c) the conventional method for 16-point, 32-point, and 64-point patterns, respectively; and (d) the wavefront orientation method for an 8-point algorithm.

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**Fig. 4.** Maps of differences in tsunami travel time (min) using numerical solutions for the ETOPO-1 grid for the Pacific Ocean; left-hand panels: differences in travel time for the conventional solutions; **(a1)** between P16 and P64; and **(b1)** between P32 and P64. Right-hand panels: differences in travel time derived using the conventional and waveform orientation methods; **(a2)** between P16 and waveform methods; **(b2)** between P32 and waveform methods; and **(c2)** between P64 and waveform methods.