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A wavefront orientation method for precise numerical determination of tsunami travel time

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Abstract

We present a highly accurate and computationally efficient method (herein, the “wavefront orientation method”) for determining the travel time of oceanic tsunamis. Based on Huygens principle, the method uses an eight-point grid-point pattern and the most recent information on the orientation of the advancing wave front to determine the time for a tsunami to travel to a specific oceanic location. The method is shown to provide improved accuracy and reduced anisotropy compared with the conventional multiple grid-point method presently in widespread use.

1 Introduction

Determining tsunami travel time is one of the fundamental roles of tsunami warning centers and other agencies tasked with estimating the possible impact of approaching tsunami waves on different coastal regions. In contrast to most other natural hazards, tsunami travel-time calculations can be performed well in advance. Because the travel time is the same when the source and the target are inverted, the inverse travel time for populated coastal sites have been computed and the corresponding databases of inverse travel-time maps created. When there is a tsunami event, the travel time to a specific coastal site is available immediately once information has been received about the initial parameters of the underwater earthquake and its location. Any initial travel time estimation will be preliminary (it is not based on the actual extent of the tsunami source) and needs to be refined as new information regarding the tsunami source becomes available.

In addition to the need for accurate real-time travel-time estimates for tsunami alerts and warnings, highly reliable estimates of tsunami travel time are required for optimizing the location of offshore tsunami warning stations used for refining information on the parameters of approaching tsunami waves (Poplavski et al., 1988). These stations require accurate advanced knowledge of the difference in wave travel time between

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specific coastal locations and the warning station (Titov et al., 2005). The greater the distance between the tsunami station and the coast, the greater the advanced forecast time for distant remotely generated tsunamis but the shorter the forecast time for locally generated tsunamis. The optimal warning system design should, therefore, be based on extensive determination of wave propagation times which, in turn, requires an efficient and accurate method for calculating the tsunami travel time. Accurate estimation of observed wave arrival times is also needed for research delineating the tsunami source region. Numerical modeling of the travel times for near-source locations can also provide important information concerning the size and shape of the source region (e.g. Abe, 1973; Fine et al., 2005; Hayashi et al., 2012). Lastly, any discrepancy between the computed and observed tsunami travel times may indicate that the geometry of the source region is incorrect or that there maybe physical effects, such as wave dispersion, nonlinearity, and coupling with elastic earthquake modes, which should be taken into account. These affects often require better wave travel time accuracy than for tsunami warning purposes.

There are presently two primary approaches for calculating tsunami travel time from gridded bathymetry: (1) by kinematic wavefront propagation calculations based on Huygens principle (Shokin et al., 1987); and (2) by solving the dynamical equations of motion, typically using finite-difference method (cf. Kowalik et al., 2005). The first method is the one most commonly used in modern tsunami travel time calculations and is favored by commercial software such as GEOWAVE (GEOWARE, 2013). Following Huygens principle, each of the points along a wave front is a source for the tsunami waves. These points serve as start locations for travel time computations to the next N neighbouring points. In GEOWAVE, N can have the values 8, 16, 32, 48 or 64, and the tsunami travel time (TTT) between points is computed using the shallow water propagating speed formula, $c = \sqrt{gh}$, where g is the gravitational acceleration and h is the water depth. Water depths are obtained from the gridded bathymetry provided by ETOPO (Amante and Eakins, 2009). If the arrival time to the nearest point is not determined, or if the previously computed arrival time is greater than the currently computed arrival time,

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the arrival time at the latter point is replaced with the newer value. At the end of the computational step, a new source point is specified as that frontal point that had the minimum travel (and arrival) times. This frontal point is converted to a permanent point to be used as source point for the next time step.

A critical step in the calculation is defining the pattern of neighbouring points. The larger the pattern, the greater the isotropy of the wave field, and the better the accuracy of the timing calculation for open ocean waves. On the other hand, a larger pattern also creates a broader region of uncertainty along the frontal zone which, in addition to the computational overhead, creates problems in zones where the water depth is changing quickly. Moreover, a larger grid pattern can cause the tsunami arrival estimations along certain angles to “jump” over shallow water areas or over islands, leading to the need to integrate the travel time over lines connecting the two points.

The wavefront orientation method proposed in this study uses a less extensive grid pattern by taking advantage of the most recently gained information on the tsunami propagation in the neighbouring grid points. In particular, the method uses knowledge about the local direction of wave front propagation obtained from previous calculations and, as a result, yields a more accurate travel time calculation without the need to apply a large pattern of grid points.

2 Methodology

2.1 The conventional method

The conventional method calculates tsunami travel time for each time step by dividing the spatial grid into three categories: (A) grid points for which the travel time has already been computed; (B) intermediate, or frontal, grid points where the travel time is to be evaluated; and (C) grid points that have yet to be reached by the advancing waves. At each time step, GEOWAVE starts the calculations from specific grid points in category A and then calculates arrival times for N neighbouring points in categories B and C

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in Fig. 2 is a linear function of the local spatial coordinates x and y , such that

$$t(x, y) = \frac{x \cos(\phi) + y \sin(\phi)}{c} + t(0, 0), \quad (5)$$

where the orientation angle, ϕ , of the tsunami wave front can be found using known values of the time difference, Δt_{OA5} , corresponding to the arrival time between points O and A5. Specifically,

$$\cos(\phi) = -c \frac{\Delta t_{OA5}}{\Delta x}, \quad (6)$$

where Δx is the incremental grid step in the x direction. The travel time to point B7 will be

$$t(\text{Point B7}) = t(\text{Point O}) + \Delta y \sqrt{\left(\frac{1}{c}\right)^2 - \left(\frac{\Delta t_{OA5}}{\Delta x}\right)^2}, \quad (7)$$

where Δy is the grid step along the y direction.

Equation (7) allows for more accurate computation of the travel time to point B7 than the conventional method. However, because it is based on previously computed travel times at two points, our method requires some initialization, which can be provided by the conventional method using the simple 8-point algorithm. Unlike the conventional method, this new approach has no fixed directional errors. As a consequence, relative errors decrease with distance from the start location (origin) of the tsunami event. This provides an improvement over the conventional method whose level of error depends on the initialization pattern.

3 Method comparison

3.1 Constant depth ocean on a spherical geographical grid

One of the primary concerns in tsunami research is determining the travel times for trans-oceanic (mostly trans-Pacific) tsunamis. It is, therefore, of interest to determine how our newly proposed method performs relative the conventional method for a spherical, constant-arc step-grid for the Pacific Ocean. We first examine the method's performance for an ocean of constant depth and then compare the results to those for an exact analytical solution. Table 3 shows the absolute errors in tsunami travel time obtained for an ocean of 4000 m depth for both methods.

According to Table 3, the maximum timing error decreases with grid pattern number, but remains measurable even for the conventional P64 algorithm. The errors are significant for the P32 and, especially, the P16 algorithms. In addition, the conventional method has a positive bias, which needs to be corrected. The GEOWARE software decreases this bias by introducing a "correction coefficient". In contrast, the proposed wavefront orientation method is almost free from errors; the maximum differences between the numerical results and the theoretical (analytical solution) values do not exceed 12 s over the entire propagation distance of 15 000 km.

Figure 3 shows the distribution of errors for the different methods. As for the plane wave case discussed in Sect. 2, the accumulated timing errors for the conventional method generally increase with distance, although the distribution of errors differs from that for the plane wave case. In particular, the errors concentrate near both sides of the latitude meridians passing through the source region but are almost absent near the parallels of longitude passing through the source region. This is because, for high latitudes, the wave propagation beams are not straight lines in the spherical coordinate system (except in the meridional direction). Specifically, the beams can't be along parallels but must be along great circles. In the near-meridional direction, the relative error is similar to that for the plane wave case; i.e. about 0.5 %, 1.3 % and 2.7 % for the conventional grid cases P64, P32 and P16. For the new wavefront model the error

It is important to note that the accuracy of our newly proposed method should not be confused with the general accuracy of tsunami travel time calculations. Actual tsunami travel times depend on many factors that are independent of the particular algorithm being used (Wessel, 2009). Specifically, actual travel time accuracy depends on the accuracy of the bathymetric dataset. In the open ocean, present gridded global datasets are sufficient for most practical applications. However, this is not the case for coastal areas. Moreover, local, high-resolution bathymetric data are not fully incorporated in the ETOPO datasets provided with the GEOWARE software. This can lead to significant errors in the travel time estimates. In addition, global datasets do not resolve narrow passes in coastal areas, requiring the use of local nested bathymetric data. The limitations of the bathymetric data are compounded by the fact that the tsunami generation region is typically poorly defined. In a scientific twist, observed tsunami travel times are often needed to better define the source region that generated the waves in the first place.

There are other physical factors which can cause the actual speeds of tsunami waves to depart from wave speed estimates obtained from simple linear shallow water theory. One of the more important factors is frequency dispersion related to non-hydrostatic effects. Non-hydrostatic effects are especially important during far field wave propagation than during near-field tsunami wave propagation, or when the tsunami source area is relatively small (shorter wavenumber waves) and/or located in relatively deep water (González and Kulikov, 1993). Although non-hydrostatic effects modify tsunami wave propagation in a depth-variable ocean, the travel times for non-hydrostatic waves can be computed in the same way as for shallow water theory by examining each wave frequency separately. This is because, for a given frequency, the arrival time is determined by the group velocity of the dispersive waves, which depends on the depth only.

In shallow water regions, frequency dispersion becomes less important. However, nonlinear effects arising from wave interaction with the tsunami wave field, and from interaction between the tsunami waves and tidal currents, can modify the travel times. In estuarine areas, tidal motions and river flow may also alter the travel times. As shown

by Watanabe (2012), it is also possible to have interactions between ocean gravity waves and elastic oscillations in Earth's mantle and between acoustic waves in the ocean. Both factors can generate noticeable changes to tsunami travel times.

Regardless of the factors affecting tsunami wave propagation, it is clear that accurate estimation of tsunami travel time is of considerable importance. Because the Huygens principle does not depend on wave speed formulations, the above factors (excluding perhaps non-linear effects) can be included in our travel time algorithm. Thus, in addition to providing improved computational efficiency and better accuracy compared to the conventional methodology, our proposed method can be used to enhance general tsunami research.

References

- Abe, K.: Tsunami and mechanism of great earthquakes, *Phys. Earth Planet. In.*, 7, 143–153, 1973.
- Amante, C. and Eakins, B. W.: ETOPO1 1 Arc-Minute Global Relief Model: Procedures, Data Sources and Analysis, NOAA Technical Memorandum NESDIS NGDC-24, 19 pp., March 2009.
- Fine, I. V., Rabinovich, A. B., and Thomson, R. E.: The dual source region for the 2004 Sumatra tsunami, *Geophys. Res. Lett.*, 32, L16602, doi:10.1029/2005GL023521, 2006.
- GEOWARE: TTT – a tsunami travel-time calculator, available at: <http://www.geoware-online.com/>, last access: 26 February 2013.
- González, F. I. and Kulikov, Ye. A.: Tsunami dispersion observed in the deep ocean, in: *Tsunamis in the World*, edited by: Tinti, S., Kluwer Academic Publisher, Dordrecht, Netherlands, 7–16, 1993.
- Hayashi, Y., Tsushima, H., and Yoshida, Y.: Real-time detection of the source area of an intensive tsunami – case study of the 2011 great east Japan earthquake, in: *Proceedings of the International Symposium on Engineering Lessons Learned from the 2011 Great East Japan Earthquake*, 1–4 March 2012, Tokyo, Japan, 2012.
- Kowalik, Z., Knight, W., Logan, T., and Whitmore, P.: Numerical modeling of the global tsunami: Indonesian tsunami of 26 December 2004, *Sci. Tsunami Haz.*, 23, 40–56, 2005.

- Poplavski, A. A., Kulikov, E. A., and Poplavskaya, L. N.: Metody i Algoritmy Avtomatizirovannogo Prognoza Tsunami, Nauka, Moscow, USSR, 127 pp., 1988 (in Russian).
- Satake, K.: Effects of bathymetry on tsunami propagation: application of ray tracing to tsunamis, *Pure Appl. Geophys.*, 126, 27–36, 1988.
- 5 Shokin, Yu. I., Chubarov, L. B., Novikov, V. A., and Sudakov, A. N.: Calculations of tsunami travel time charts in the Pacific Ocean – models, algorithms, techniques, results, *Sci. Tsunami Haz.*, 5, 85–122, 1987.
- Titov, V. V., González, F. I., Bernard, E. N., Eble, M. C., Mofield, H. O., Newman, J. C., and
 10 Venturato, A. J.: Real-time tsunami forecasting: challenges and solutions, *Nat. Hazard*, 35, 41–58, 2005.
- Wessel, P.: Analysis of observed and predicted tsunami travel times for the Pacific and Indian Oceans, *Pure Appl. Geophys.*, 166, 301–324, doi:10.1007/s00024-008-0437-2, 2009.

Table 1. Directional error parameters for the conventional method (denoted by the letter “P”) applied to a plane square grid. RMS is the root mean square of the error and STD is the standard deviation of the errors. Column 2 gives the angle yielding the maximum error.

Method	Angle β_{\max} , degrees	Maximum error (%)	Mean error (%)	RMS (%)	STD (%)
P8	22.5000	8.239	5.477	6.006	2.4624
P16	13.2825	2.749	1.438	1.6581	0.8260
P32	9.2175	1.308	0.490	0.6442	0.4184
P48	7.0181	0.755	0.245	0.3325	0.2250
P64	5.6550	0.489	0.149	0.2054	0.1417

Table 2. Relative errors for the proposed wavefront orientation method and a plane square grid. Distance from the source location at Point O (0,0) is measured in terms of the specific point number on the grid. The larger the grid number, the greater the distance from the source region. RMS is the root mean square of the error and STD is the standard deviation of the errors.

Average distance from the source, points	Maximum error (%)	Mean error (%)	RMS (%)	STD (%)
96–104	0.303	0.188	0.210	0.093
496–504	0.091	0.056	0.062	0.027
996–1004	0.052	0.032	0.036	0.015
4996–5004	0.013	0.018	0.010	0.003

Table 3. Errors in tsunami travel time for a constant (4000 m) depth ocean with bathymetry gridded at 1-arc minute steps for both the conventional and proposed methods. Values are compared to exact values obtained from an analytical solution. Results for the conventional method are denoted with a “P”; those for the waveform method with an “F”. RMS is the root mean square of the error and STD is the standard deviation of the errors.

Method	Minimum error (min)	Maximum error (min)	Mean error (min)	RMS (min)	STD (min)
P16	0	17.0	7.1	8.2	4.0
P32	0	8.5	3.1	3.7	1.9
P64	0	4.2	1.6	1.8	0.8
F8	-0.1	0.2	0.1	0.1	0.03

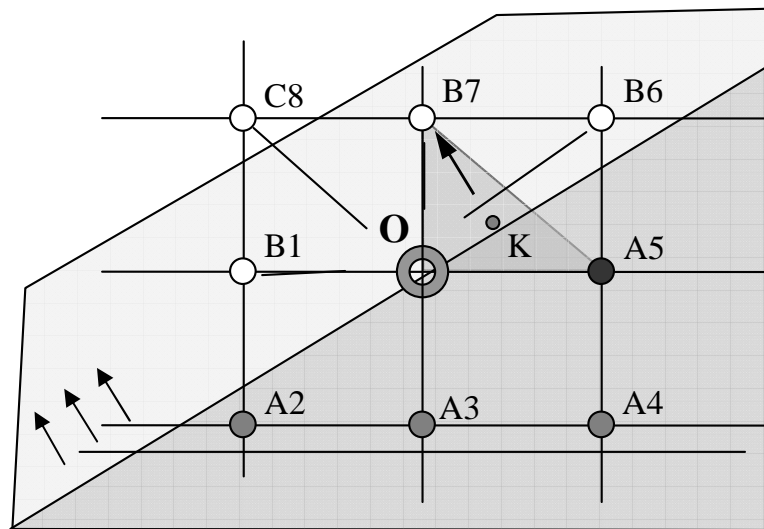


Fig. 2. Schematic showing the 8-point pattern used in the proposed wavefront orientation method. The shading is the same as for the conventional method presented in Fig. 1. The three small arrows show the direction of wave front propagation.

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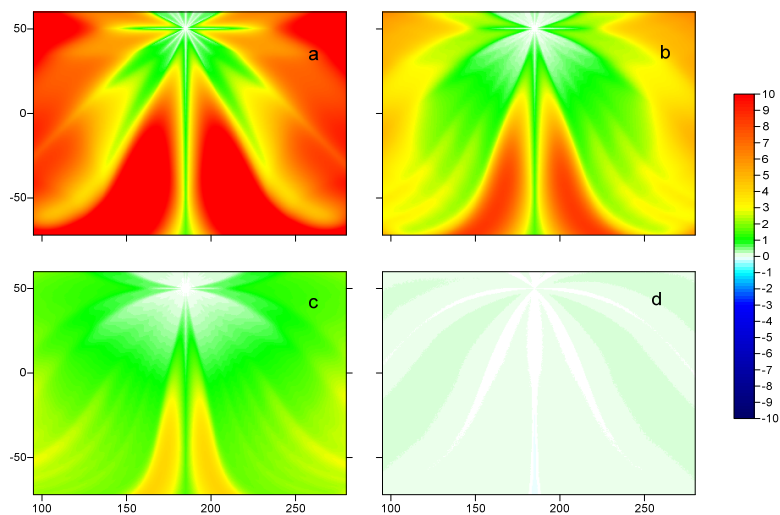


Fig. 3. Maps of tsunami travel time errors (min) for a uniform depth (4000 m) ocean using: (a–c) the conventional method for 16-point, 32-point, and 64-point patterns, respectively; and (d) the wavefront orientation method for an 8-point algorithm.

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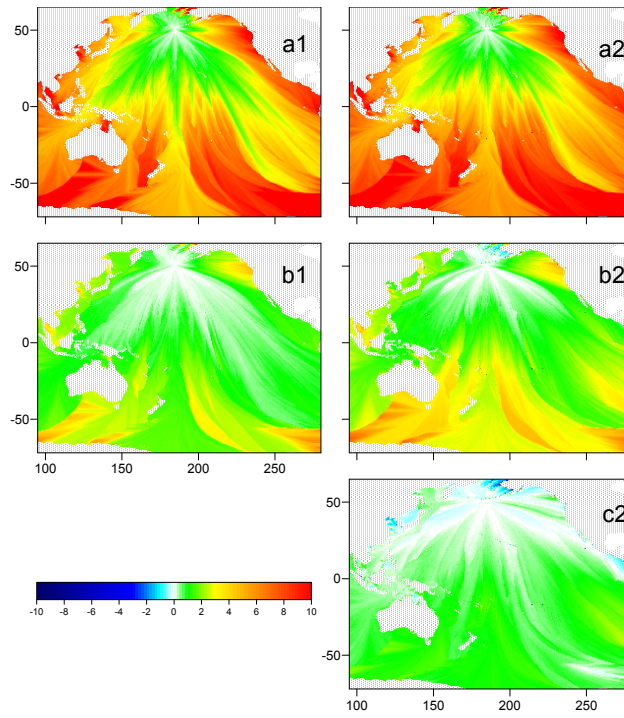


Fig. 4. Maps of differences in tsunami travel time (min) using numerical solutions for the ETOPO-1 grid for the Pacific Ocean; left-hand panels: differences in travel time for the conventional solutions; **(a1)** between P16 and P64; and **(b1)** between P32 and P64. Right-hand panels: differences in travel time derived using the conventional and waveform orientation methods; **(a2)** between P16 and waveform methods; **(b2)** between P32 and waveform methods; and **(c2)** between P64 and waveform methods.